

EFFECT OF FINITE EXPOSURE SLITS IN DETERMINATION OF THE LINE SPREAD FUNCTION AND MODULATION TRANSFER FUNCTION

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The use of modulation transfer functions (MTF) is now well established as a tool to evaluate the imaging performance of radiologic imaging systems (MORGAN 1962, MORGAN et coll. 1967, ROSSMAN 1964).

The technique most generally used in such a procedure is that of first obtaining the line spread function (LSF) of the imaging system and performing a Fourier transformation utilizing a digital computer. This method is well described in the literature, (MORGAN et coll., ROSSMAN). In practice the LSF is obtained by exposing the imaging system under test through a jaw system which defines a very narrow slit of roentgen rays, usually 10 μm wide for film-screen systems. Theoretically the slit used should be infinitesimally narrow, however, the slits used in practice have some finite width and serve only as an approximation to a true infinitesimally narrow slit. The actual width size finally used represents a compromise necessitated by the requirement of allowing enough radiant energy through the slit to obtain a meaningful measurement. A loss of accuracy in determining the LSF is associated with use of such a finite slit. In the past, this loss of accuracy has been minimized by choosing a slit width satisfying the condition that no significant LSF changes occur if slit width is made either just narrower or just wider. Using this approach exposure slit widths 10 μm wide have, for instance, become standard for analysing radiographic intensifying screen type systems (MORGAN et coll., ROSSMAN et coll. 1964). The choice of

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slit width has been based on experimental results and there has been no definitive theoretical description of the error introduced into the LSF when finite slit widths are used. The purpose of this report is to derive an expression for the theoretical MTF of a finite slit width, and to indicate a procedure which may be used to correct LSF data obtained when a finite slit is utilized.

Past attempts to quantitate finite slit effects

A number of attempts to quantitate the actual effect on the finite slit on the determination of the LSF have been made. Most notable of these is the work of ROSSMAN et coll. They assumed that the LSF of a typical radiographic screen-film system could be expressed by an exponential model of the form

$$A(x) = (a/2) \exp(-a|x|) \quad (1)$$

where $a = 0.01 \mu\text{m}$ for an average system and x is the spatial variable in μm . They then calculated that for a $10 \mu\text{m}$ wide slit the inherent error at the maximum of the spread function was about 2.5 per cent and about 0.04 per cent outside the slit width. The limitation of their approach consisted in the assumption of a mathematical model for the LSF. No such model for the LSF will fit all imaging systems and slit configurations. Because of the difficulty in finding such mathematical models most attempts at analyzing the effect of a finite slit width on the LSF have had to remain empirical. The experimental approach has been to determine which slit size has no significant effect on the determination of the LSF. In general it has been observed that the poorer the imaging system the larger the slit width which can be used. As an example, in the imaging of nuclides with focused collimators (a relatively poor imaging system) line sources 1 mm and larger are common, while for radiographic non-screen films, slit widths as narrow as $3 \mu\text{m}$ must be used. Radiographic screen systems as mentioned previously are typically analyzed with $10 \mu\text{m}$ wide slits. The considerations in the following sections are aimed at determining the effect in terms of MTF theory of using a finite slit width on any imaging system, independent of the imaging quality of that system, and independent of any postulated mathematical models fitting that system.

MTF of a finite exposure slit width

To determine the MTF of a finite exposure slit the following well known result is invoked:

$$g(x) * h(x) \xrightarrow{\text{FT}} G(\omega) \cdot H(\omega) \quad (2)$$

Where $g(x)$ and $h(x)$ are two functions with spatial variable x while $G(\omega)$ and $H(\omega)$ are their Fourier transforms (FT) in terms of frequency ω ($\omega = 2\pi f$). This result

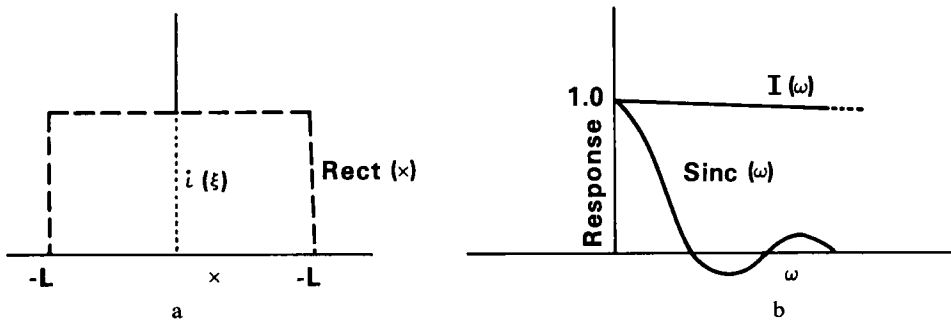


Fig. 1. Finite rectangular exposure slit transform pair. a) Typical line exposure distribution in the spatial domain given by the function Rect (x) as compared to ideal slit impulse $i(\xi)$. b) Distribution in frequency given by sinc (ξ) and $I(\omega)$ as obtained by Fourier transformation of Rect (x) and $i(\xi)$, respectively. Composite response of product is just sinc (ω) since $I(\omega) = 1$ for all ω .

states that the convolution of two functions in the spatial domain (given by star symbol) is the simple product of their respective Fourier transforms in frequency space.

In Fig. 1 a appears an ideal roentgen line exposure distribution or impulse $i(\xi)$ in the spatial domain as might be formed by an infinitesimally narrow or ideal exposure slit. The ideal line input is merely an infinitesimally thin spike centered at $x = 0$, and is analogous to the Dirac delta function of line impulse. A graph of its Fourier transform $I(\omega)$ appears in Fig. 1 b. $I(\omega)$ consists of a unit height line in the frequency domain for all ω from zero to infinity. This essentially states that an ideal line impulse represents a signal having present all frequencies with equal amplitudes. In contrast with the ideal situation, a finite slit is used in practice. Such a slit will have some width, assumed here to be of width $2L$. In this case the resultant roentgen exposure formed by such a slit will consist of a rectangular roentgen pulse of width $2L$ (Fig. 1 a). This rectangular pulse, which will be denoted by Rect (x), may also be viewed as the convolution of a line impulse $i(\xi)$, over the width $2L$ to which the slit had been increased. The Fourier transform of Rect (x) is given by the function sinc (ω) (GABEL & ROBERTS 1973). If the Fourier transform of $i(\xi)$ is $I(\omega)$, then eq. (2) becomes:

$$i(\xi) * \text{Rect}(x) \xrightarrow{\text{FT}} I(\omega) \text{ sinc}(\omega) \tag{3}$$

i.e. the convolution of the function $i(\xi)$ and Rect (x) forms a Fourier transform pair with the product of their respective transforms. The transform of an infinitesimally thin line impulse, such as $i(\xi)$, is unity for all ω , i.e. $I(\omega) = 1$, eq. (3) then becomes:

$$i(\xi) * \text{Rect}(x) \xrightarrow{\text{FT}} \text{sinc}(\omega) \tag{4}$$

The function sinc (ω) in eq. (4) resulted from the spatial convolution of a line impulse and a rectangular pulse and represents the frequency content resulting from

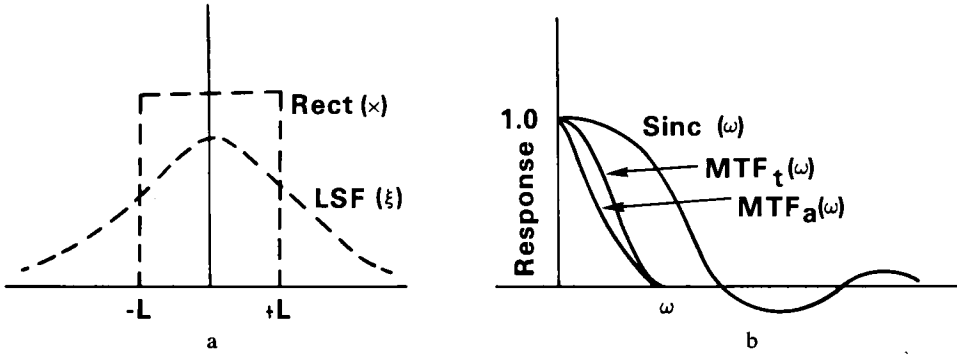


Fig. 2. Finite rectangular exposure slit serving as input into a real (non-perfect) imaging system. a) Exposure distribution $Rect(x)$ serving as input into a system with resulting apparent LSF (ξ) at output. b) Frequency distribution (Fourier transforms) of LSF (ξ) and $Rect(x)$ given by the apparent measured $MTF_a(\omega)$, and $sinc(\omega)$, respectively. The true system MTF is given by $MTF_T(\omega) = MTF_a(\omega)/sinc(\omega)$.

such a convolution. This essentially means that the finite exposure slit serves as a filter and allows through only those frequencies with amplitude given by eq. (4). This distribution of frequencies serves as the input to the imaging system under test when a finite width slit is used. The function, $sinc(\omega)$, appears frequently in imaging theory and is of the general form (GARBEL & ROBERTS 1973)

$$sinc(\omega) = \frac{\sin 2\pi fa}{2\pi fa} \tag{5}$$

where a is a constant, f = spatial frequency variable and $\omega = 2\pi f$. Graphically the function $sinc(\omega)$ is illustrated in Fig. 1 b with periodic fluctuations of decreasing amplitude about the x -axis.

The output from an imaging system which has a line input as described will now be discussed. In general the rectangular pulse input experiences a broadening or smearing beyond its original borders when incident on the imaging system. This is due to the inherent lack of the system to reproduce the incident signal faithfully. If input had been an ideal line exposure, the resulting distribution would be defined as the system (LSF). The result observed with a finite slit width is the convolution of the LSF with the slit function $Rect(x)$. These two functions are illustrated in Fig. 2 a. Since the Fourier transform of the LSF (ξ) and $Rect(x)$ are $MTF(\omega)$ and $Sinc(\omega)$, respectively, eq. (3) may be rewritten as follows when substituting $i(\xi) = LSF(\xi)$ and $I(\omega) = MTF(\omega)$

$$LSF(\xi) * Rect(x) \xrightarrow{FT} MTF(\omega) sinc(\omega) \tag{6}$$

The left hand side of eq. (6) represents the convolution of the true system LSF with the rectangular slit function. The right hand side of the equation indicates the resultant or observed output frequency function expressed as the product of the sinc function

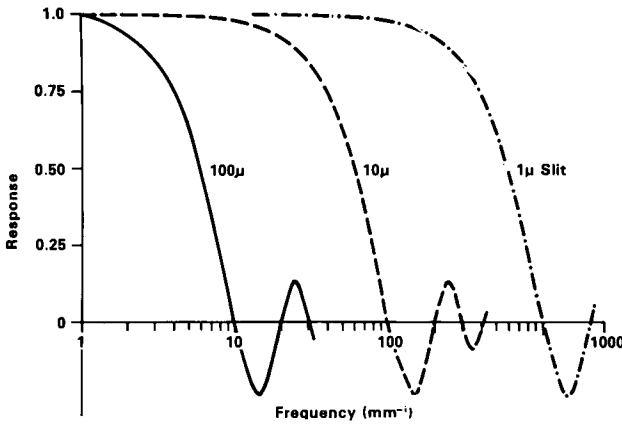


Fig. 3. MTF of finite rectangular exposure slits of overall width 1, 10 and 100 μm .

and the true system MTF, which now may be written $\text{MTF}_T(\omega)$. This product is what is obtained experimentally as the apparent $\text{MTF}_a(\omega)$. With these modifications eq. (6) then becomes:

$$\text{MTF}_a(\omega) = \text{MTF}_T(\omega) \cdot \text{sinc}(\omega) \tag{7}$$

or

$$\text{MTF}_T(\omega) = \text{MTF}_a(\omega) / \text{sinc}(\omega) \tag{8}$$

The relationship between these functions in the frequency domain is given in Fig. 2 b.

From the preceding discussion the physical significance of the LSF approach in the determination of the MTF may now be appreciated. A line impulse containing all frequencies with equal amplitude is fed into the system. The system degrades the line impulse and the LSF results. The Fourier transform is taken of the LSF, however the Fourier transform determines the sinusoidal frequency content of the LSF. That frequency content represents the amplitude of those specific frequencies which have been allowed to pass through the system. The MTF is defined as the ratio of the output amplitude to the input amplitude. The input amplitude is unity, therefore the MTF is equal to the output amplitude or just simply the Fourier transform of the LSF. If a $\text{rect}(x)$ junction is used rather than the ideal line impulse, then the input amplitude at any given frequency is not unity but in fact is given by the sinc function and the MTF is given by ratio of amplitudes given in eq. (8). The denominator of eq. (8) can be termed the MTF of the rectangular finite exposure slit, $\text{MTF}_s(\omega)$, that is

$$\text{MTF}_s(\omega) = \text{sinc}(\omega) \tag{9}$$

Specifically, for a slit half width of L and a spatial frequency $f = \omega/2\pi$, the slit MTF is written as:

$$\text{MTF}_s(f) = \frac{\sin 2\pi fL}{2\pi fL} \tag{10}$$

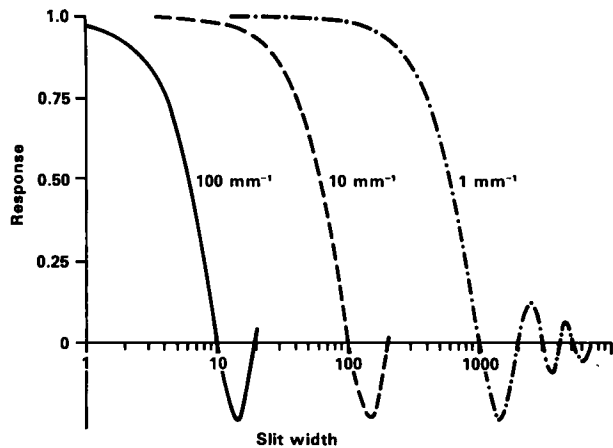


Fig. 4. Amplitude response at particular frequencies as function of slit widths in μm .

This function is plotted in Fig. 3 for slit width = $100 \mu\text{m}$, $10 \mu\text{m}$ and $1 \mu\text{m}$. In each case these functions fluctuate about zero out to infinity with consecutively lower amplitudes.

Fig. 4 displays the slit MTF from another aspect, when particular frequencies for different size slit widths are considered. Thus, the loss of response or degradation of the MTF may be determined by using this curve for a particular frequency or band of frequencies as perhaps described by some effective frequency.

Of particular interest is the curve for slit width of $10 \mu\text{m}$ ($L = 5 \mu\text{m}$) as stated before, $10 \mu\text{m}$ has been the standard slit width used for the determination of the MTF of radiographic intensifying screen systems. These type systems yield responses (MORGAN et coll.) within the frequency band of 0 to 10 mm^{-1} . Fig. 4 indicates that the value of the exposure slit MTF is approximately unity for this frequency band. Consequently no correction is needed for intensifying screen systems when a slit of $10 \mu\text{m}$ is used. This is in confirmation with experimental results. If a $100 \mu\text{m}$ slit were to be used, as was in fact used by some in the early days of the application of the LSF concept, it is seen in Fig. 3 that the MTF of this width slit is significant for frequencies about 1.5 mm^{-1} and explains some of the discrepancies in the early results. As a further example of the use of these relationships, MORGAN has shown that the MTF of non-screen imaging systems decreases to 0.1 at frequencies of 50 to 60 mm^{-1} . Such an imaging system would require an exposure slit width 1 to $5 \mu\text{m}$ wide. Widths of $3 \mu\text{m}$ have in fact been found to be optimum for these type of systems.

From Fig. 3 the following rule may be derived to use for the determination of the required slit width K in μm if the frequency band width of the imaging system in question ranges up to a value of B in mm^{-1} and a loss of response no greater than 2 per cent will be tolerated at any frequency

$$K = 100/B \quad (11)$$

Therefore if $B = 10 \text{ mm}^{-1}$, as found in typical intensifying screen systems, then $K = 10 \text{ } \mu\text{m}$ and a loss of no greater than 2 per cent will be expected at any frequency. Similar rules for 5 and 10 per cent may also be invoked (VILLAFANA 1975).

Correction for the slit MTF

Eq. (8) may be used to determine the true system MTF from experimental data obtained using a finite rectangular exposure slit. To accomplish this correction, the apparent or observed MTF is divided by the value of eq. (10). This effect corrects the observed MTF point by point in the frequency domain. There is one practical limitation to this approach, however, and that is the fact that the slit MTF does have zero points. At these, and at near zero points, eq. (8) approaches infinity. This in effect rules out the possibility of using relatively large slit widths with the hope of correction later.

In eq. (8) $\text{sinc}(\omega)$ represents the input amplitude at any frequency and $\text{MTF}_a(\omega)$ represents the output amplitude at any frequency. Therefore, the ratio of eq. (8) is really the basic definition of the system MTF as the fractional reduction in amplitude at any frequency. Eq. (8) may then be generalized to any input slit configuration as follows:

$$\text{MTF}_T(\omega) = \text{MTF}_a(\omega)/B(\omega) \quad (12)$$

where $B(\omega)$ is the Fourier transform of some slit function $b(x)$ which can be referred to as the slit transmittance function. The function $b(x)$ will represent the spatial intensity distribution of the roentgen beam after transmission through the slit configuration in question. Therefore, the true MTF of an imaging system can be obtained by applying any known input signal $b(x)$ and correcting the observed MTF by simple division with the transform of the known slit transmittance function. The frequency domain corollary of this is that any known frequency distribution may be used as the input to a system under test. Examples of this might for instance be Gaussian or exponentially distributed signals. However, it should be observed that in practice the simplest choice for input signal is in fact the finite rectangular slit function.

In nuclear medicine it is usual to use a small diameter capillary tube as a line source. Correction for this diameter may be made as illustrated in the discussion assuming a rectangular configuration for the source as a first approximation. If accuracy warrants it, actual cross sectional distribution across the tube can be calculated and its Fourier transform be used as $B(\omega)$ in eq. (12).

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SUMMARY

The modulation transfer function of radiologic imaging systems is commonly obtained by determining the line spread function (LSF) of the system and computing its Fourier transform. Ideally, LSF should be obtained with infinitesimally narrow slits. The use of finite slits for obtaining the LSF is analyzed theoretically. An expression for the MTF of a finite rectangular slit is derived. Slit width correction of observed MTF is discussed as well as the correction for any generalized slit configuration.

ZUSAMMENFASSUNG

Die Modulations-Übertragungsfunktion des röntgenologischen Bildsystems lässt sich im allgemeinen durch eine Bestimmung der Linienstreuungsfunktion (LSF) des Systems und Berechnung von dessen Fourier-Transformation erhalten. Im Idealfall sollte die LSF mit unendlich nahen Schlitzen erhalten werden. Die Verwendung von endlichen Schlitzen, um die LSF zu erhalten, wird theoretisch analysiert. Ein Ausdruck für MTF eines endlichen rechteckigen Schlitzes wird hergeleitet. Die Korrektur der Schlitzweite der beobachteten MTF wird diskutiert ebenso wie die Korrektur für jegliche generalisierte Schlitzkonfigurationen.

RÉSUMÉ

La fonction de transfert de modulation des systèmes d'imagerie radiologique est habituellement obtenue en déterminant la fonction de dispersion linéaire (LSF) du système et en calculant sa transformée de Fourier. Dans l'idéal, la LSF devrait être obtenue avec des fentes infiniment étroites. L'emploi de fentes finies pour obtenir la LSF est étudié théoriquement. L'auteur en déduit une expression de la fonction de modulation de transfert d'une fente rectangulaire finie. Il examine la correction de la MTF observée en fonction de la largeur de la fente ainsi que sa correction pour toute les configurations généralisées de la fente.

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