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A RHEOLOGIC MODEL FOR CORTICAL BONE

A Study of the Physical Properties of Human Femoral Samples

BY

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CHAPTER I

INTRODUCTION

The study of the physical properties of bone has been a subject of interest for more than a century, the first contribution being attributed to *Wertheim* (1847). The considerable body of information which has accumulated during this time has been reviewed in recent years by *Evans* (1957) and *Knese* (1958). These surveys reflect that there are diverse opinions concerning the reaction of bone to mechanical forces. This diversity is, in part, due to the fact that cortical bone is an heterogeneous or anisotropic material, a feature which has been described in the works of previous authors. (*Rauber*, 1876; *Maj* and *Toaiari*, 1937; *Carothers et al*, 1949; *Evans*, 1952; *Dempster* and *Liddicoat*, 1952; *Evans* and *Lissner*, 1959; *Smith* and *Walmsley*, 1959; *Yamada* and *Motoshima*, 1960; *Dempster* and *Coleman*, 1961, *Yoshikawa et al*, 1963; *McElhaney et al*, 1964; *Sedlin* and *Hirsch*, 1965.)

Almost all previous studies have been directed toward the quantification of one or more physical properties. Although many physical properties have been described or indicated for bone, few attempts (*Currey*, 1964; *Mack*, 1964) have been made to explain the total behavior of bone as a material. No attempt has been made to integrate this spectrum of properties into a matrix for the further study of the reaction of bone to mechanical forces. One approach to integration is to develop a model in which the behavior of bone can be simulated by mathematical analogues and which can be modified and extended as new information is derived.

For this purpose, the author felt that the approach of the rheological field of physics offered potential as its concepts had been useful in the study of many materials and structures. (*Reiner*, 1960; *Stulen*, 1962; *Papazian*, 1962; *Johns* and *Wright*, 1964.) Rheology is concerned with the deformation and flow of materials and a rheologic model which represents bone does not involve the use of anatomic components. With a model, one might ultimately simulate experiments that are difficult to perform. The application of this approach is the central theme of the thesis.

In order to obtain valid information that could be used in the development of a model, it was necessary to standardize the source material and testing methods. The standardization required detailed study of the methods of preparing, storing, and testing specimens in order to ascertain the effects of multiple variables upon the results. Since human post-mortem material was easily available at this institution, it was elected to use autopsy bone.

This eliminated the problem of a sporadic source from biopsy and amputation material. The femur was chosen for study since it is a representative long bone capable of yielding large numbers of samples.

Quantitative data under defined *in vitro* conditions were obtained in the course of study. These are not presented for their own sake, but are used in the demonstration of qualitative effects of variates for the standardization of methods and in the development for the model.

CHAPTER II

MATERIAL AND METHODS

Source of material

This study was based upon 663 samples of human femoral cortex obtained from 43 autopsy subjects ranging in age from 14—91 years. The postmortem examination and specimen sampling was performed between 7 and 96 hours after death. Specimens were obtained as the needs of each experiment dictated, in an attempt to minimize the time between sampling and testing. Although the effort was made to limit the sampling to previously normal subjects (e.g. trauma cases), it was only infrequently possible to obtain bone from such subjects at the time needed. The subjects utilized for specimen source material thus represent a random selection of autopsy material available at this institution. The vital and morbid data for all subjects used in both quantitative and qualitative analyses in this study are listed in Table 1.

Site of sampling

Samples were prepared from mid-femoral diaphysis for 75 per cent of the experiments reported here, with upper and lower diaphysis providing material for the remainder. This distribution is noted in Figure 1; and it is seen that all samples for bending tests and some tension tests came from mid-femoral diaphysis, with tension and compression samples coming from upper and lower diaphysis.

Organization of material

The experiments, with subjects contributing specimens, are listed in Table 2. At the time of all experiments, no information relative to the subject was available to the author save autopsy code number and age, thereby resulting in "blind" type of testing. One group of experiments was planned to determine if changes being studied were present during most of adult life. For these experiments, ten subjects ranging from 19—64 years of age were grouped. Twelve specimens were then prepared from each mid-femur and site-matched samples from all subjects were grouped, resulting in twelve groups of ten specimens from ten subjects. (Figure 2 A) These groups are referred to as Series Ten in the remainder of the text.

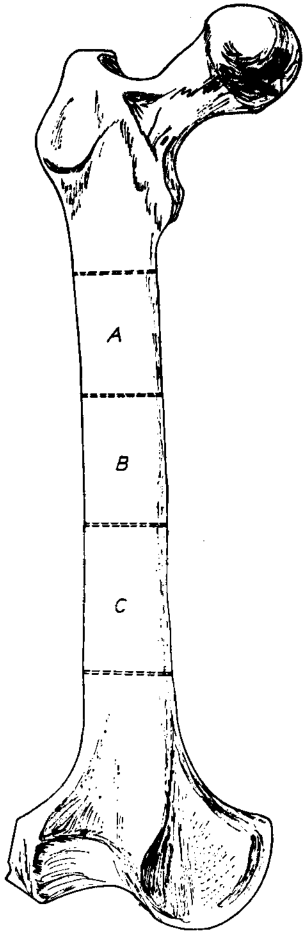


Figure 1. Site of Selection of Samples. The femur is divided into three zones, **A**, **B**, and **C**. Tension samples generally came from Zone **A**. All bending samples came from Zone **B**. Zone **B** was also used *in toto* for some compression tests. Zones **C** and **A** provided source material for compression blocks.

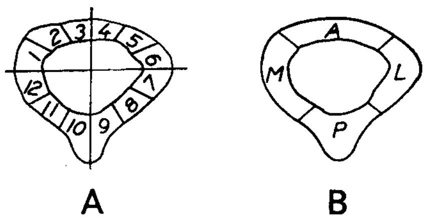


Figure 2. Cross-section of Femoral Diaphysis. **A.** The site of sampling for the Series Ten. Twelve bending samples were prepared for each subject in the group, and then corresponding samples for the ten subjects were studied as one test group. **B.** Indicates Anterior, Lateral, Posterior and Medial quadrants respectively, for site coding of other samples with respect to quadrant of origin.

TABLE 1. *The Study Group.*

Subject Number	Sahlgren Hospital Number	Age, years Sex	Date of Death	Time Between Death and Sampling-Hours[1]	Cause of Death	Underlying Disease or Condition	Type of Samples Prepared[2]	Experiments in which Samples used[3]
1	501219	14 F	16-XII-64	44	Cardiac Surgery	Congenital Heart Disease	T	19, 20, 21, 22
2	490822	15 F	18-XII-64	94	Cerebral Hemorrhage	Aneurysm post. cerebral art.	T	19, 20, 21, 22
3	460108	19 F	2-II-65	34	Hepatic Failure	Chronic lupoid hepatitis	T	19, 20, 21, 22
4	460718	19 M	7-IV-65	48	Malignant Lymphogranuloma		CB	Series Ten
5	360326	29 M	16-I-65	31	Cerebral Tumor		CB	Series Ten
6	330523[4]	32 F	23-XI-64	70	Librium Poisoning	Suicide	LC, SC	19, 21
7	330103	32 F	20-II-65	36	Electric Shock Therapy	Post-partum psychosis (2 wks)	T	19, 20, 21, 22
8	291206	34 M	25-III-64	7	Chronic Renal Disease	Diabetes Mellitus	BB, T	17, 19
9	300106	34 M	7-VIII-64	20	Suffocation	Epilepsy	BB, T	13, 17, 18, 19
10	310626	34 F	28-III-65	42	Amniotic Fuid Embolization	Caesarian Section	CB	Series Ten
11	230501	41 F	17-VI-64	12	Carcinoma Breast	Diffuse Metastases	BB	4, 10, 17, 19
12	231026	41 F	7-III-65	47	Cerebral Malacia	Essential Hypertension	LC, SC	19, 21
13	230112	42 M	7-III-65	39	Carcinoma Pancreas	Hepatic Spread; Laparotomy	CB	Series Ten
14	180921	46 F	13-VII-64	29	Uremia	Multiple myeloma	BB	4, 17, 18, 19
15	171230	47 F	20-X-64	27	Malignant Melanoma		BB	1, 17, 19
16	170823	47 M	27-X-64	41	Cerebral Hemorrhage, Uremia	Diabetes Mellitus	BB	21
17	170325	48 M	24-IV-65	60	Acute tracheobronchitis		CB	Series Ten
18	150420	50 M	9-III-65	18	Uremia	Chronic Pyelonephritis	LC, SC	12, 19
19	141118	50 M	24-II-65	48	Reticulum cell sarcoma nasopharynx	Hepatic, vertebral spread	T	19, 20, 21, 22
20	121111	52 M	31-I-65	52	Pulmonary embolus	Chronic bronchitis, emphysema	CB	Series Ten
21	100616	53 M	2-III-65	53	Cardiac Failure	Emphysema, chronic bronchitis	LC, SC	12, 19
22	120608	53 M	4-III-65	33	Acute pancreatitis	Chronic Alcoholism	LC, SC	19, 21
23	090621	55 M	23-VII-64	32	Bronchopneumonia	Cachexia	BB	4, 17, 18, 19
24	100118	55 F	25-I-65	9	Hepatic, pulmonary, abscess	Carcinoma stomach	CB	Series Ten
25	071210	56 M	9-VI-64	48	Cerebral Hemorrhage	Hepatoma	BB	4, 10, 17, 19
26	060628	59 M	5-III-65	22	Myocardial Infarction		CB	Series Ten
27	030802	61 M	9-II-65	45	Hypernephroma	Diffuse spread	CB	6
28	020828	62 M	16-VIII-64	91	Bronchopneumonia	Schizophrenia	BB, T	1, 17, 18, 19
29	021029	62 F	2-III-65	24	Intestinal Infarction		BB	16
30	020915	62 M	11-II-65	32	Congestive Heart Failure	Empyema, Atherosclerosis	CB	Series Ten
31	000902	64 M	11-XI-64	36	Carcinoma Stomach	Diffuse spread	BB	21
32	990813	65 F	12-II-65	60	Cardiac Failure	Rheumatic Heart Disease	CB	Series Ten
33	980313	66 F	21-VIII-64	90	Bronchopneumonia		CB	7, 11
34	—[5]	67 M	-VIII-64	—	Massive Trauma	Auto accident	BB, T	13, 17, 19
35	970523	68 F	17-I-65	36	Carcinoma Breast	Diffuse spread	CB	20, 21
36	950220	69 M	2-V-64	60	Myocardial Infarction		BB, T	17, 19
37	960330	69 M	6-II-65	48	Lymphatic Leukemia		T	22
38	901128	74 F	13-X-64	18	Myocardial Infarction	Severe Atherosclerosis	BB	17, 19
39	881002	75 F	13-V-64	30	Pulmonary Embolus		BB, T	13, 17, 19
40	880812	76 F	27-X-64	39	Pulmonary Embolus	Carcinoma ovary, Osteoporosis	BB	21
41	870403	78 F	11-X-64	33	Gastrointestinal Hemorrhage	Hepatic cirrhosis, Esophageal varices	BB	1, 17, 19
42	810626	83 M	17-XII-64	24	Appendicitis	Arteriosclerosis, pulmonary edema	T	2, 19
43	731213	91 M	14-VI-64	65	Myocardial Infarction	Pyelonephritis	BB	4, 10, 17, 19

[1] All times are ± 2 hours.

[2] CB = cantilever bending.
BB = Supported beam, center load.
LC = Complete cross-section of diaphysis.
SC = Small compression block.
T = Tension.

[3] See Table 2 for code.

[4] Vasa Hospital case.

[5] No further data available on this subject.

TABLE 2. *The Experiments and Studies*

Experiment Number	Description of Procedure	Page	Number of Specimens Used	Subjects Contributing
	The Effects of:			
1	Freezing	20	74	15, 28, 41
2	Formalin	21	10	42
3	Alcohol	22	55[1]	Series Ten
4	Short-term air-drying, end-supported samples	24	101	11, 14, 23, 25, 43
5	Short-term air-drying, cantilever samples	26	10	Series Ten
6	Five-day air drying and rewetting	27	12	27
7	Four-day heat-drying and rewetting	28	10	33
8	Short term wet heat	28	10	Series Ten
9	Short term dry heat	29	10	Series Ten
10	Four day heat-drying	29	12	11, 25, 43
11	Heat upon stress-relaxation, hysteresis	30	10[1]	33
12	Grinding upon bone temperature	32	5	18, 21
13	Temperature changes during testing to failure	34	77	9, 34, 39
14	Temperature changes upon deflection of cantilever samples	36	10	Series Ten
15	Repetitive loading, cantilever samples	37	30	Series Ten
16	Repetitive loading, end-supported samples	37	11	29
17	Variation in properties in different femoral quadrants	38	336[1]	8, 9, 11, 14, 15, 23, 25, 28, 34, 36, 38, 39, 41, 43
18	Correlation of porosity and properties	41	125[1]	9, 14, 23, 28
19	Demonstration of stress-relaxation in different axes, subjects	52	71	Series Ten, 1, 2, 3, 6, 7, 12, 18, 19, 21, 22, 42, + Gp. 17
20	The effects of different rates of deformation	54	27	Series Ten, 1, 2, 3, 7, 19, 35
21	Demonstration of hysteresis in different axes, subjects	55	65[1]	1, 2, 3, 6, 7, 12, 16, 19, 22, 31, 35, 40, Series Ten
22	Deformation under a constant load	56	17	1, 2, 3, 7, 19, 37
	Total Specimens Used		663[2]	

[1] Specimens utilized in toto or in part in other procedures.
[2] Net total of all specimens, duplication accounted for.

Seventy-eight samples were utilized in more than one test. In these, it was felt that prior testing would not interfere with the study that was under consideration.

Choice of size of specimens

In studying the physical properties of bone, some authors have used intact bones or large samples of bone, (*Bell et al*, 1941; *Marique*, 1945; *Carothers et al*, 1949; *Weir et al*, 1949; *Evans*, 1952; *Hirsch* and *Brodetti*, 1956; *Lissner* and *Evans*, 1956; *Evans et al*, 1958; *Lease* and *Evans*, 1959; *Vose* and *Kubala*, 1959; *Hirsch* and *Frankel*, 1960; *Frankel*, 1960; *Evans et al*, 1962) while others have used small samples. (*Wertheim*, 1847; *Rauber*, 1876; *Olivo et al*, 1937; *Dempster* and *Liddicoat*, 1952; *Evans*, 1952, 1957, 1961; *Knese et al*, 1955; *Smith, J. W.* and *Walmsley*, 1957; *Amprino*, 1958; *Currey*, 1959; *Hollinghaus*, 1959; *Bassett* and *Becker*, 1962; *Taysum et al*, 1962; *Vose*, 1962; *Ascenzi* and *Bonucci*, 1964; *Mack*, 1964; *McElhaney*, 1964; *Hirsch* and *Evans*, 1965; *Sedlin* and *Hirsch*, 1965; *Smith, R. W.* and *Keiper*, 1965). Utilization of intact bones can minimize the time between obtaining the sample and actual testing and obviates some of the questions that arise from the use of small samples, i.e. are the properties affected by removal from location in the cortex? However, intact bones can vary greatly in their dimensions with resultant changes in some physical constants (e.g. moment of inertia) from place to place, (*Koch*, 1917; *Knese et al*, 1955), with the result that it becomes difficult to reduce data to a standard basis for comparison (e.g. Kp/mm^2) of results. The use of a small sample enables one to normalize data, but incurs the potential objection concerning the removal of the sample from its location. However, there is some evidence that data from small samples are representative of the behavior of larger segments of a long bone. (*Carothers et al*, 1949). On balance this evidence, coupled with the number of tests required, led to the use of small specimens in this study.

Preparation of specimens

After removal from the body, segments of femoral diaphysis were worked fresh or stored at $-20°$ C until such time as they could be worked. The general method of preparation consisted of sectioning the femur in its long axis with a handsaw or bandsaw under a constant cold aqueous drip. These strips were labelled according to their quadrant of origin (Fig. 2 B) and could be further divided in their long axis. The segments were then reduced to final size utilizing *underwater* machine and manual grinding. *During preparation, except when a specimen was actually being handled, it was kept in Ringer's* *solution at room temperature*. Special shaping tools that were used in the reduction of longitudinal strips have been described elsewhere (*Sedlin* and *Hirsch*, 1965).

Specimen size and shape

The size of specimens utilized in testing was varied several times during the conduct of this group of experiments, with the type of test being performed being the chief determinant. A specimen for a bending test in which both ends were supported and a central load applied usually measured 2 mm broad, 1 mm thick and 30—40 mm long. Occasional specimens measuring $1 \times 1 \times 30$ mm or $2 \times 2 \times 40$ mm were used in testing the effects of size or when extra or fewer samples were desired. Specimens which were bent as cantilevers measured $1.7-2 \times 1.7-2 \times 50$ mm in dimensions. There was some variation in the size of bending specimens from lot to lot.

A specimen for Series Ten was 3.5 mm^2 in cross-section dimensions with a moment of inertia of 1 mm^4.

Specimens for tension testing consisted of rectangular planks with a reduced central area. The standard size of specimens used in tests reported here was 50 mm long, 2 mm thick, 5 mm broad at the ends and 3 mm broad in the central reduced area, which was 25 mm long. The zone of transition from wide to reduced area was an arc. 1.5 mm at either end.

Specimens that were tested in compression were of two types: 1) A complete cross-section of mid-femoral diaphysis, 5 cm in length with ends which had been milled flat and parallel under 96% alcohol drip. 2) A cube $5 \times 10 \times 15$ mm which was first milled to approximate final size under 96% alcohol drip and subsequently ground under water.

After preparation of a specimen was completed, the cross-section dimensions were determined with a friction release micrometer accurate to .005 mm. All specimens with more than 1.5% variation in cross-section dimensions were discarded or reworked. After measurements were completed, the specimen was ready for testing. If possible, the specimen was tested at this time. Otherwise, it was placed in Ringer's solution and stored at $-20°$ C until testing.

Testing equipment

The tests described in this study were performed on an Instron TT-CM floor model tensile test machine (Figure 3) or with equipment described subsequently. This machine has a capacity that enables one to test specimens under many conditions of loading, (i.e. tension, compression, shear, bending) through a wide range of loads and a wide range of rates of constant deforma-

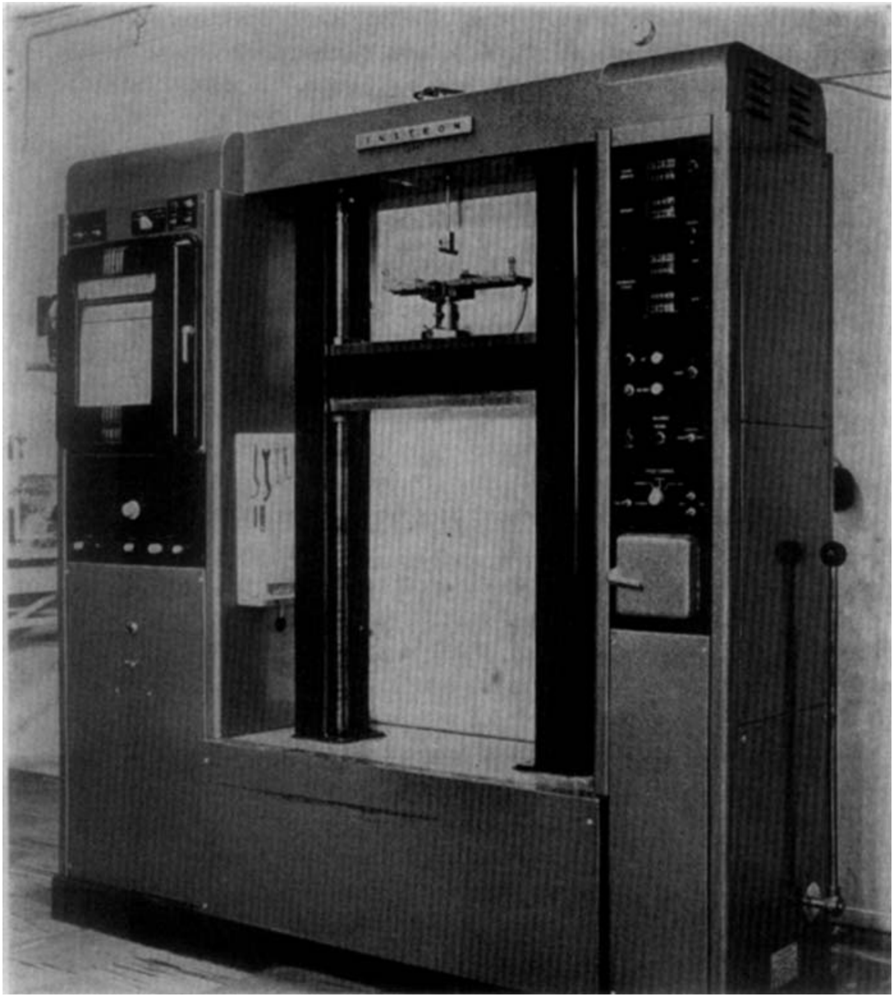


Figure 3. Instron Floor Model Tensile Test Machine. Machine is shown with bending platform attached to cross-bar. At left the recorder is seen, and on the right, various controls and gauges.

tion. The load is applied via exchangeable cross-heads (Figure 4), and load range is determined by interchangeable load cells with capacities ranging from 2 grams to 5,000 kg. The rate of cross-head movement can be varied from .02 cm/min up to 50 cm/min. The speed of the recorder paper can be varied independently of the speed of cross-head travel (rate of deformation) in order to magnify certain portions of the load-deformation diagram. Recording is via a servo-controlled pen permitting a direct-write, load-deformation

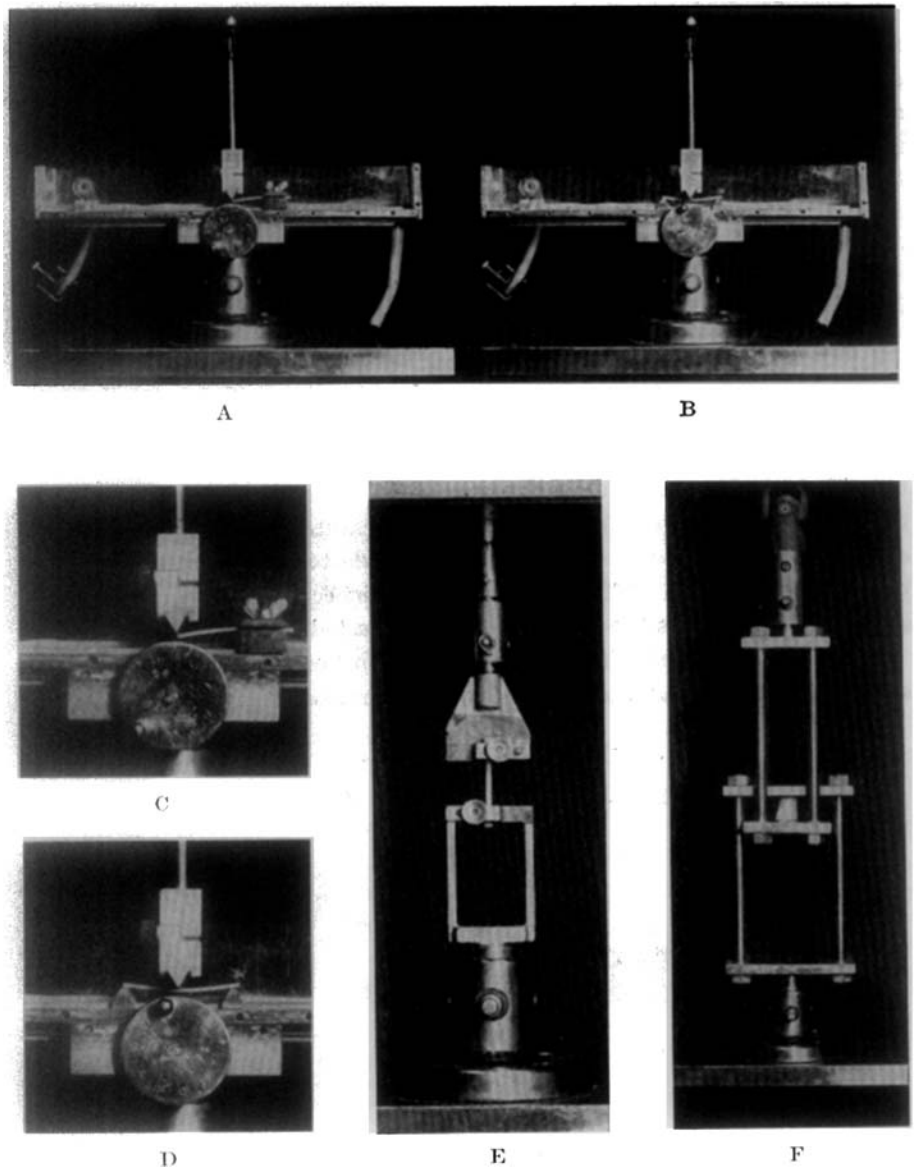


Figure 4. Loading heads for Instron used in this Study. A & B. The bending platform with front face removed. Dangling tubes connect to water pump with heating element. In A, a specimen is being bent as a cantilever. In B, another specimen is being bent as an end-supported center loaded beam. C & D. Closeups of specimen in A and B respectively. E. Set-up for tension testing. F. The compression cage with a 5 cm segment of distal diaphysis. The upper half of the cage is stationary while the cross-bar moves downward carrying the lower half of the cage.

diagram. The scales of the load-deformation diagram (Figure 5) were determined by the load cell being used, and the ratio of recorder speed to speed of cross-head travel. If desired, one can run the recorder while cross-heads are stopped, thus obtaining a direct record of the behavior of stress over time under a constant deformation. The machine can also be made to cycle between two loads or two extensions enabling the observer to study deformation occurring between fixed loads, or stress with known deformations. All operations are accurate to $\pm 0.5\%$.

By use of several different types of loading heads, compression, cantilever bending, simple beam and tension tests were done. The set-ups for testing are shown in Figure 4.

This machine is designed for tests requiring a constant rate of deformation or modifications. It is not possible to apply an absolutely constant load or a constant rate of loading. It is possible to provide a constant load $\pm 1\%$, but then, one must rely upon readings of the scale of the machine to determine deformation over time. An additional feature evolves from the speed of the recorder pen, which requires 1.5 seconds to travel full scale. Deformations occurring in stiff materials may be inaccurately recorded if small loads are used, unless the specimen size is adjusted for this feature.

In view of the necessity for absolutely constant load during the creep studies (page 56) and the desire for direct measurements of deformation,

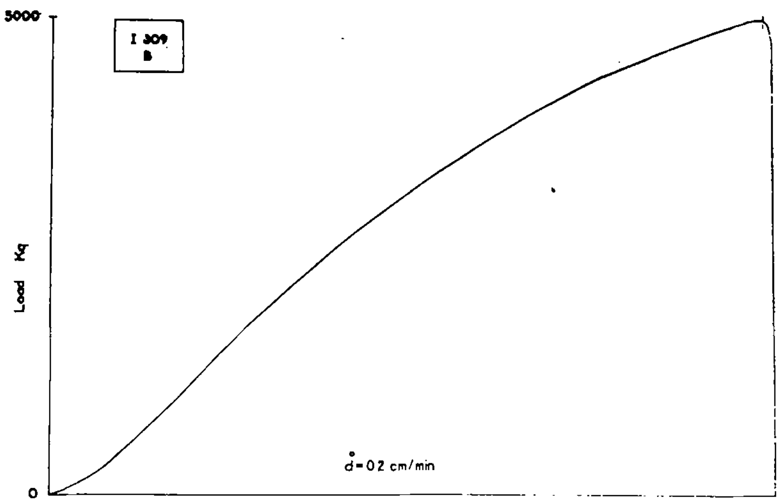
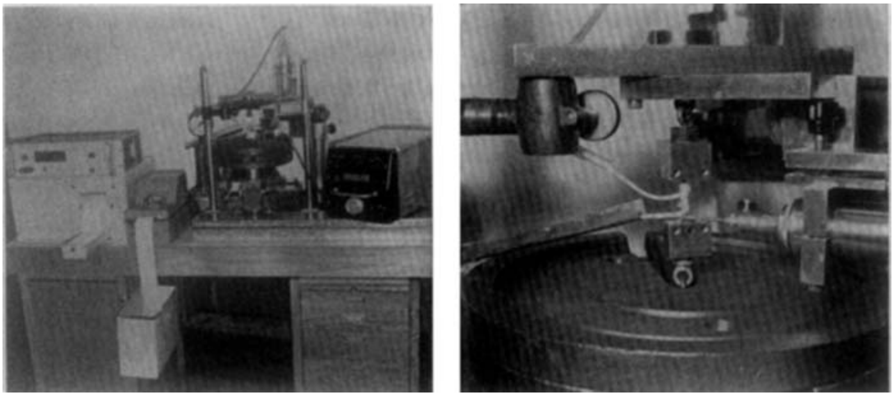


Figure 5. A load deformation diagram of a compression test to failure performed on a 5 cm segment of mid-diaphysis. Failure is indicated by the vertical at top of diagram. Subject identification is at upper left of chart. The rate of deformation is indicated by $\dot{d}$.



A B

Figure 6. Apparatus used in creep tests. A. Pictured from the left are voltmeter with printing readout; recorder; pump; loading assembly with bottle of Ringer's solution at top; amplifier. B. Close-up of loading apparatus. The specimen is in the center wrapped in a wick that is seen to come from a polyethylene tube at left and above. The tube is connected to a bottle of Ringer's solution. An initial siphon action soaks the wick and the specimen stays moist throughout the test. The rope is led off into a trough to prevent fluid collection during the test. At the right of the specimen, the extension pickup can be seen. When the weights are released, the pick-up registers the distance that the lower specimen clamp travels.

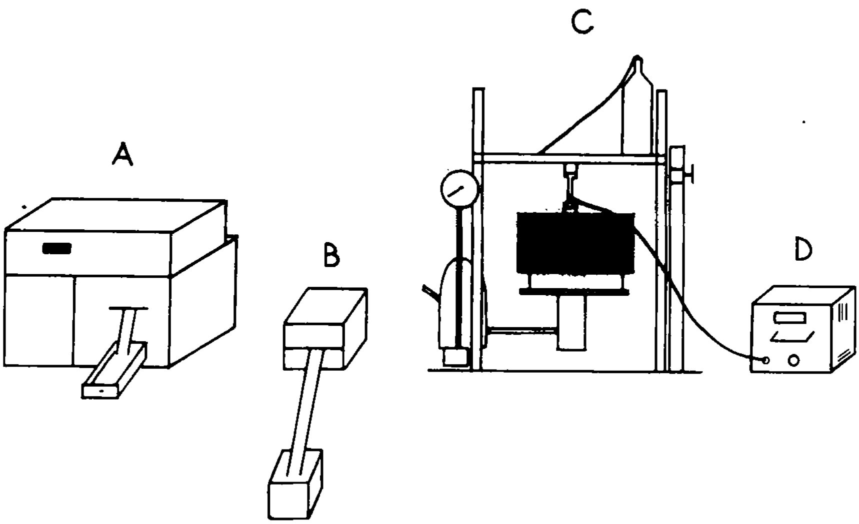


Figure 7. Diagram of creep-test apparatus. A. Voltmeter with printing readout. B. Recorder. C. Test assembly. Weights are indicated in black. The pump is at the left of the assembly. The loading-unloading are performed by pumping the weights up to unload or down to load. Specimen and bottle are as in Figure 6. D. The amplifier with connection to extension pick-up.

a different apparatus was used for the creep tests. This is pictured in Figures 6, 7. The equipment consisted of a Taylor-Hobson pickup, amplifier, and recorder, plus a Hewlett-Packard digital voltmeter with printing read-out. The voltmeter permitted readings of deformation accurate to 0.5μ while the recorder provided a direct deformation-time curve over a 250μ scale (Figure 23, page 58).

Equipment for the determination of heat conduction in bone is described with the experiment.

Methods of testing

The majority of the tests in this study were bending, either as a cantilever or as a centrally loaded, end-supported beam (Table 2). The supports for the specimens in bending were provided by a special loading head for the Instron which had been designed to function as a clasimeter or cantilever load support. This loading head has detachable sides, enabling one to test under fluid or in air (Figure 8, 9). All bending tests that were not done to determine the effects of drying were performed in Ringer's solution at either room temperature or 37° C. The temperature was controlled $\pm 1°$ and fluid exchanged via a cyclotherm pump.

All tests, except for the creep and heat measurement studies, were performed in a special room with controlled temperature and relative humidity at 21° C$\pm$1° and 66%$\pm$2% respectively. These were constantly monitored. Prior to testing, all specimens were heated to 37° C in Ringer's solution for one hour in a constant temperature bath unless the specific requirements of the test dictated room temperature and/or hydration.

Compression specimens were tested in a modification of the Instron compression cage (Figure 4 F). It required 15—30 seconds to properly position the specimen in the cage for a compression test.

Tension specimens were first tested utilizing specially designed holding clamps which were lined with silicon carbide paper to prevent slipping (Figure 4 D). Subsequently a fine groove pattern was cut on the faces of the clamps and it was found that this worked as well as the silicon carbide paper in preventing slipping, with the clamps holding loads up to 100 kg in some cases. The groove pattern was preferred since this eliminated the need for changing sandpaper. This type of clamp was improved in the creep studies equipment, in that one face was morticed to receive a specimen, which simplified the procedure of accurate alignment of the specimens. The tension specimens required approximately 2—3 minutes to position and carefully align in the vertical axis of the load.

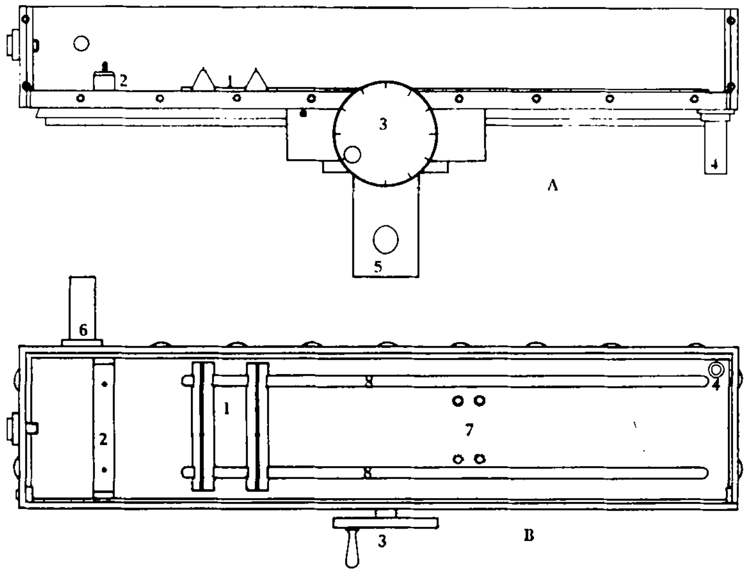


Figure 8. Diagram of bending platform. **A.** Front view showing supports for specimen (1). Note the tiny studs that project from the top of supports. These aid in alignment of the specimen. (2) Support for baffle to reduce fluid turbulence. (3) Control handle to shift entire platform (4). Exit port for fluid (5) Stand for connection to machine. **B.** Top view showing (6) Entrance port for fluid, (7) Holes for anchoring of cantilever attachment (8) Track on which supports (1) can be shifted.

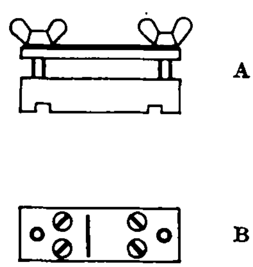


Figure 9. Diagram of cantilever attachment for the bending platform. **A.** Front view showing slots for track. **B.** Top view of lower half of attachment showing an alignment wire that facilitates seating of the specimen.

If not otherwise stated all tests indicated in the text are bending tests to failure performed with unfixed end supports and progressive central load at a rate of 1 cm/min. Specimens for tension which were tested to failure were deformed at rates of 1—5 mm/min. More rapid rates were not feasible since the recorder speed was too slow to record deformation in the usual samples used. The usual rate of deformation for compression tests was 5 mm/min.

Statistical methods

Standard statistical methods were utilized in evaluating the results. (*Snedecor*, 1946). Appropriate method was utilized for specific experiment. Thus, where a specimen was tested, treated and tested again (as on pages 22, 27) the differences before and after treatment were treated appropriately and the Student t test was used in determining probability. Where two groups of samples from the same subject or from several subjects were compared, variance analysis was employed, and the F-ratio used to determine probabilities. Where two variates were being tested on groups of samples from different patients, the variance was partitioned utilizing two-way variance analysis. Such was done in determining the significance of differences of various locations in the femur. Product-moment correlation was used in evaluation of relations between data from microscopic analysis of some samples and their physical properties and in the correlation of different physical properties in samples. The 5% level of probability was chosen as the criterion of statistical significance.

Formulae used in calculation of physical properties

Bending

Stress was calculated from the equation $\sigma = Mc/I$ (*Roark*, 1954) where $M = PL$ for cantilever bending and $PL/4$ for end-supported beam bending with center load. $I = BD^3/12$, $c = D/2$.

Therefore, in cantilever bending,

$$\sigma = \frac{PLc}{I} = \frac{6PL}{BD^2} \text{ Kp/mm}^2$$

In end-supported, centrally loaded beams,

$$\sigma = \frac{PL/4c}{I} = \frac{3/2PL}{BD^2} \text{ Kp/mm}^2$$

The modulus of elasticity, E, was calculated from the formulae $E = PL^3/48Iy$ Kp/mm^2 for an end-supported beam and for a cantilever $E = PL^3/3Iy$ Kp/mm^2, where P and y were determined from the most linear portion of a load-deformation diagram. If such could not be determined, it was calculated from a tangent drawn at a point representing 25% of the estimated breaking load.

Tension

$\sigma = P/A$ where A is the original cross section area at the fracture site and $E = \sigma/\epsilon$ where P and ϵ were measured from the most linear portions of the load deformation diagram, or calculated from a portion of the curve at an estimated 25% of the ultimate strength.

Other

Energy absorption in bending was determined by planimetric measurement of the area under the load-deformation diagram, and appropriate conversion utilizing load, paper speed and specimen size.

Notation

σ = Stress
ϵ = Unit strain
P = Load
I = Moment of inertia
E = Modulus of elasticity
L = Original length of specimen or test area
M = Maximum moment
A = Cross-section area of specimen or test area
B = Breadth of specimen
D = Depth of specimen
c = Distance from neutral axis to fiber of greatest stress
y = Deflection in bending

CHAPTER III

SOME FACTORS THAT EFFECT THE PHYSICAL PROPERTIES OF BONE

The following features incident to the preparation, and evaluation of physical property data from cortical bone samples are considered in this chapter:

The effects of methods of storage

The effects of varying the conditions of preparation and testing

The effect of repetitive loading

The influences of site of sampling, age, and microanatomy

An interim summary is presented at the end of the chapter.

The effects of methods of storage

Since it is apparent that some time must elapse between obtaining a specimen and its testing, the possibility exists that a change takes place in the physical properties during this period. If so, one must either test the specimen quickly after obtaining the sample or preserve it in order to minimize the change.

Of the three common methods of preserving specimens, only fixation with formalin has received much attention. No systematic studies of the effects of freezing and alcohol have been performed. *Frankel* (1960), noted no difference in the breaking strength of femurs that had been frozen. *Carothers et al* (1949) found that embalming increased the bending strength in rat femurs. *Calabrisi* and *Smith* (1951) stated that embalming for 40—60 days decreased the compressive strength of human tibial cortical samples about 13%. *McElhaney et al* (1964) studied the effect of embalming on beef femoral samples and found that there was a significant 12% reduction in the compressive strength but very little upon tensile strength, maximum strain, and modulus of elasticity. *Evans* (1964) showed that embalming increased the tensile strength of human tibial samples. With this information as background, the following experiments were done.

Several properties after freezing

Seventy-four specimens, either 2×1 mm or 2×2 mm in cross section dimensions, from subjects 15, 28, & 41 were prepared as rapidly as possible after obtaining the bone. Forty-three were tested three hours after removal from the body, and thirty-one were frozen at

$-20°$ C for 3 to 4 weeks. These were tested after thawing and heating to 37° C. Care was taken to site-match specimens in order to have representative samples from all quadrants present in both groups. The results are summarized in Table 3. There is an unequal number of specimens in the two groups due to the inability to obtain equal numbers of specimens from all quadrants of the three femurs.

TABLE 3. *The Effect of Freezing upon some Physical Properties of Bone*[1]

Condition	No. of samples	Ultimate Fiber Stress Kp/mm^2	Modulus of Elasticity $Kp/mm^2 \times 10^3$	Energy Absorbed to failure $Kpcm/mm^3 \times 10^{-2}$	Total Deflection to failure mm
Fresh	43	16.7 ± 3.0	$1.41 \pm .24$	9.16 ± 2.72	$1.81 \pm .64$
Frozen	31	17.6 ± 3.2	$1.39 \pm .23$[2]	9.67 ± 3.05[2]	$1.60 \pm .53$[2]
Probability		$> .05$	$> .05$	$> .05$	$> .05$

[1] — All values presented for a physical property are mean values with one standard deviation indicated by $\pm$.

[2] — Only 23 specimens are represented in these groups due to some difficulties with recording apparatus. Valid maximum stress values could be derived, but other properties could not.

It can be seen that freezing and subsequent heating to 37° C had no significant effect upon the physical properties in the combined data. Also there were no significant differences between fresh and fresh-frozen specimens in each of the individuals. These observations, coupled with those of *Frankel*, lead to the conclusion that the physical properties of bone are not altered by freezing at $-20°$ C.

Modulus of elasticity after formalin fixation

Ten tension specimens from subject 42 were prepared fresh, the filet size in these samples being 2×2 mm and 20 mm long. They were tested at room temperature without prior warming. They were deformed at a rate of 2 mm/minute until a load of 8—9 kg was reached. This load approximated a unit stress of 2—2.3 kp/mm^2. The specimens were unloaded and placed in 10% formalin solution for three weeks and then retested. Tangent moduli of elasticity were calculated and the differences were analyzed for each specimen.

The average E value for the samples in the fresh state was 400 ± 54

Kp/mm^2*. After three weeks in formalin solution, it was 417 ± 41 Kp/mm^2. This 4.3% increase in the modulus of elasticity was suggestive, but not significant ($.10 > p > .05$).

The findings are in agreement with those reported by *McElhaney et al* (1964) as regards the tensile deformation. In view of the reports cited earlier, it appears that formalin or embalming solutions may affect some properties and not others. Accordingly, it was felt that this means of preservation should not be used.

Deformation after alcohol fixation

Fifty-five specimens from Series Ten set-up as cantilevers were used in two experiments. All had been frozen prior to testing. Most had been been previously tested in checking other variables. In the first experiment, 30 of the specimens were thawed at room temperature and placed in 40% alcohol for 10 days and tested. Then they were placed in Ringer's solution for 3 hours and retested. For the second experiment, 25 specimens were thawed and tested at room temperature in Ringer's solution. Then they were placed in 40% alcohol for 5 days and tested again, after which they were replaced in Ringer's solution for 24 hours and retested.

Specimens were deformed at a rate of 1 cm/min up to a load of 100 grams in every test. This load resulted in an average maximum bending moment of 3 Kpmm and a resultant average maximum stress of 3 Kp/mm^2. Deflection that occurred at 100 grams load was determined for each test.

The results were as follows: In the first group, 24 showed less deformation in alcohol than in Ringer's solution. The average decrease was .03 mm, representing a change of 4%. This difference was significant beyond the 1% point ($t_{29} = 3.12$). For the group of 25 specimens, the mean deflection in the first test was .76 mm; after 5 days in alcohol, it was .74 mm; and after replacement in Ringer's solution, it was .73 mm. The 2.5% decrease in deformation after 5 days in alcohol was significant beyond the 1% level ($t_{24} = 3.307$). But unlike the first group, these specimens failed to show an increase in deformation after being replaced in Ringer's solution.

The effects of immersion in Ringer's after soaking in alcohol, observed in the first experiment, could not be duplicated in the second phase of the second experiment. Inasmuch as it could not be stated that the effects of alcohol were reversible, it was not employed as a preservative in the study.

* One standard deviation. If not otherwise indicated, all mean values are precented with one standard deviatiation in the remainder of the text.

The effects of varying the conditions of preparation and testing

In the course of specimen preparation and testing, several operations introduce factors which could influence results. Experiments in this section examine the effects of four of the possible variables — air drying, heat exposure, temperature changes during grinding, and temperature variations at the time of testing.

The effect of air drying

The effects of air drying on physical properties have been more frequently discussed than the other factors noted above. *Wertheim* (1847) showed that stress and strain in dry bone were more nearly proportional than in fresh samples. *Rauber* (1876) found that drying increased the tensile and compressive strength of human femoral samples. *Bell et al* (1941) stated that drying in air produced no significant alterations in the bending and torsional strength of intact rat femora. *Evans* and *Lebow* (1951) showed that air-drying increased the modulus of elasticity, hardness and tensile strength of human femoral samples, but decreased the shearing strength and energy absorbing capacity. *Dempster* and *Liddicoat* (1952) found that drying increased the tensile strength of cortical samples and confirmed Wertheim's finding that the stress-strain curve to failure is more nearly linear in dry bone. *Smith* and *Walmsley* (1959) showed a 7% increase in the modulus of elasticity after one hour of air drying in one human sample. They also showed that progressive loss of weight of a bone sample occurred upon removal from the body. In addition, they showed that longitudinal contraction occurred progressively up to five hours in drying another sample of bone. *Lease* and *Evans* (1959) noted a decrease in the fatigue life of some metatarsals and an increase in the fatigue life of others associated with drying. *Dempster* and *Coleman* (1961) found increases in the ultimate tensile and bending strength of dry tibial samples both parallel and at right angles to the long axis of the osteons. *Evans* (1964), in another study, showed that drying increased the tensile strength of femoral samples. *Ascenzi* and *Bonucci* (1964) showed that drying increased the tensile strength of single osteons.

The weight of the evidence indicates that dry bone is stronger and deforms less than wet bone. Implicit in several of the studies is the probability that some changes due to air drying are reversible. With the exception of the studies of *Amprino* (1958) and *Smith* and *Walmsley* (1959), little work has been done relative to the reversibility of the drying process. Since specimens in this study were kept wet during preparation and were tested wet, it would appear of little moment to test this feature. However, to determine the effect of drying upon physical properties not previously studied and to confirm earlier work, the following studies were performed.

Several properties after variable periods of air drying

One hundred thirty-seven samples from subjects 11, 14, 23, 25, and 43 were prepared, but 12 specimens were used in the heat experiment described on page 29. Insofar as possible, the specimen groups consisted of one specimen from each quadrant of each subject's femur. Thus, a basic group consisted of 20 specimens. Specimens from 4 of the subjects were 2×1 mm in cross-section dimensions, while specimens from the fifth subject were 2×2 mm. All specimens were 30 mm in length and tested to failure as centrally loaded end-supported

TABLE 4. *The effect of air drying upon ultimate fiber stress in bending*[1]

Time in air	Subjects					Average
	11	14	23	25	43	
Wet	21.3	21.2	19.2	19.2	13.4	18.9
15′	24.6	19.2	17.5	20.9	17.1	19.9
30′	19.1	22.8	20.5	22.1	19.2	20.7
45′	22.2	23.3	20.9	22.7	19.3	21.7
60′	26.0	26.2	19.4	24.0	19.5	23.0
Average	22.6	22.5	19.5	21.8	17.7	20.8

Analysis of Variance

Source of Variation	Degrees Freedom	Sum of Squares	Mean Square
Total	24	197.25	8.22
Subjects	4	93.35	23.34
Time in air	4	51.74	12.94
Residual	16	52.16	3.26

$F_{4,16}$ subjects $= 7.16$, $p < .01$

$F_{4,16}$ air time $= 3.97$, $.05 > p > .01$

[1] All values are mean values for all samples from a subject for a given state of time in air. Comparison of individual values would have necessitated the introduction of a third variate — location within the femur. Units are Kp/mm^2.

TABLE 5. *The effect of air-drying upon the modulus of elasticity in bending*[1]

Time in air	Subjects					Average
	11	14	23	25	43	
Wet	1.85	1.97	1.57	1.76	1.28	1.69
15′	1.90	1.44	1.30	1.66	1.38	1.54
30′	1.60	1.65	1.38	1.64	1.47	1.55
45′	1.58	1.73	1.31	1.62	1.42	1.53
60′	1.56	1.72	1.28	1.70	1.37	1.52
Average	1.70	1.70	1.37	1.68	1.38	1.57

Analysis of Variance

Source of Variation	Degrees Freedom	Sum of Squares	Mean Square
Total Variation	24	.9412	0.392
Subjects	4	.6017	.1504
Time in air	4	.0919	.0230
Residual	16	.2476	.0155

$F_{4,16}$ subjects $= 9.703$, $p < .01$

$F_{4,16}$ time in air $= 1.484$, $p > .05$

[1] Values are stated in units of Kp/mm$^2 \times 10^3$. Derivation of values as in Table 4.

beams at a deformation rate of 1 cm/min. Span-thickness ratios were 20:1 in the smaller specimens and 15:1 in the larger specimens. All specimens were tested at room temperature. One group was tested submerged, and other groups were tested after being air-dried for periods of 15′, 30′, 45′ and 60′ respectively. Twenty-five specimens were tested after air-drying of five or ten minutes. The ultimate fiber stress, tangent modulus of elasticity, energy absorbed to failure and total deflection to failure were calculated. These results are partially summarized in Tables 4, 5, and 6.

It can be seen from Tables 4—6 that there were significant differences between subjects for each of the properties that were measured. The only significant change related to time in air was the increase in ultimate fiber stress. The analysis for total deflection to failure was similar to that of elastic modulus and energy absorption. Significant differences could be demonstrated between subjects, but not between various states of hydration. The specimens tested after five or ten minutes of air drying did not show any changes.

TABLE 6. *The effect of air-drying upon energy absorbed to failure in bending*[1].

Time in air	Subjects					Average
	11	14	23	25	43	
Wet	12.7	11.3	7.45	8.28	6.11	9.17
15′	16.3	9.43	6.90	9.39	8.40	10.1
30′	11.5	10.9	7.89	9.01	7.66	9.39
45′	11.1	9.85	8.27	10.6	12.4	10.4
60′	13.2	14.7	6.82	11.4	13.2	11.9
Average	13.0	11.2	7.47	9.74	9.55	10.2

Analysis of Variance

Source of Variation	Degrees Freedom	Sum of Squares	Mean Square
Total	24	.01640	.000684
Subjects	4	.00839	.0021
Time in Air	4	.00228	.000670
Residual	16	.00575	.000359

$F_{4 \cdot 16}$ subjects $= 5.850$; $p < .01$

$F_{4 \cdot 16}$ time in air $= 1.588$; $p > .05$

[1] Units are Kp cm/mm$^2 \times 10^{-2}$.
Units in variance analysis are original.

Deformation after one hour of air drying

One group from Series Ten was tested as cantilevers at a length of 30 mm to a load of 100 grams in room temperature Ringer's solution. They were dried for an hour and retested. Deflection that occurred at 100 grams was measured in both tests and the differences in individual specimens compared.

An increase in deflection of the specimens was demonstrated after drying. The average deflection of the wet tests was .79 mm; that of one-hour drying tests was .89 mm. The differences in individual specimens were significant on inspection. The specimen size was determined after drying for one hour and, although slight shrinkage occurred, it was not enough to account for the change. Thus, the modulus of elasticity of this group decreased after one hour of air drying.

Deformation after five days of air drying

Twelve $2 \times 2 \times 50$ mm specimens from subject 27 were tested with a beam length of 30 mm at room temperature in Ringer's solution. Then, they were placed between glass slides under constant pressure from rubber bands to reduce warping. These "sandwiches" were dried under negative pressure at room temperature for five days. The specimens were remeasured and retested. They were then replaced in Ringer's solution for 24 hours, remeasured and tested a third time. Specimens were loaded to 250 grams in each test at a rate of 1 cm/min. Deflection at this load was determined. It was necessary to calculate the modulus of elasticity in these tests, since shrinkage sufficient to decrease the moments of inertia by 10% occurred after drying. This shrinkage was fully reversed by the 24 hour replacement in Ringer's solution. Differences in individual specimens were evaluated.

The results of these tests are as follows: The mean modulus of elasticity in the initial test was $1.26 \pm .15 \times 10^3$ Kp/mm^2. After drying for five days at room temperature, the modulus changed to $1.65 \pm .17 \times 10^3$ Kp/mm^2. After reimmersion in Ringer's solution, it was $1.21 \pm .14 \times 10^3$ Kp/mm^2. The increase in modulus of elasticity after drying was significant ($t_{11}=17.17$). In addition, the 5% decrease in modulus between the first and third tests was significant ($t_{11}=3.157$).

The effect of heat

Several workers have provided a background for this section. *Rauber* (1876) noted that the tensile and compressive strength of bone were decreased by heating. *Amprino* (1958) demonstrated a progressive, consistent increase in microhardness in bone that had been subjected to increasing temperatures from 38°—120° C. Irreversible changes in some mechanical properties of rat tail tendon have been shown to occur above 40° C (*Rigby et al*, 1959). It is also known that the properties of collagen are altered at temperatures around 60° C (*Neustadt* and *Rotstein*, 1963). *Marique* (1945) demonstrated the presence of hysteresis in bone and *Smith* and *Walmsley* (1957) stated that stress relaxation was present in bone; but the effects of drying and temperature changes on these properties have not been considered. In addition, there is no information available on the relationship between heat and bending. In this section, five experiments involving heat and bending are presented.

Deformation after four days of dry heat

Ten specimens measuring $2 \times 2 \times 50$ mm, from subject 33, were tested fresh in Ringer's solution at room temperature. Afterwards, each was placed in a glass "sandwich" and dried for four days at 105° C. Then they were retested dry, replaced in Ringer's solution for 24 hours and tested again. The specimens were loaded as cantilevers 30 mm in length to 250 grams at a rate of 0.5 cm/min. After measuring the deformation in the three instances, tangent moduli of elasticity were calculated for every specimen in each condition. Since shrinkage sufficient to change the average moment of inertia by 13% occurred after heat drying, it was necessary to evaluate E instead of deflection. Differences in individual specimens were compared.

The mean modulus of elasticity in the initial test was $1.34 \pm .14 \times 10^3$ Kp/mm². In the second test, this value was $1.56 \pm .24 \times 10^3$ Kp/mm². After rewetting, it became $.91 \pm .15 \times 10^3$ Kp/mm². The differences in values for individual specimens were significant on inspection. Testing of the differences showed $t_9 = 7.515$ for the change due to heat drying; $t_9 = 7.836$ for the decrease in deformation after rewetting; $t_9 = 11.447$ in the comparison of the original test and the rewet test.

Deformation after one hour of wet heat

This study utilized one group from Series Ten which was tested as cantilevers with a length of 30 mm. After immersion in Ringer's solution at room temperature, the specimens were loaded to 100 grams. Subsequently, they were boiled in Ringer's solution for one hour, replaced in room temperature Ringer's solution for 30 minutes and then retested. Next, they were put in the refrigerator for 24 hours and tested again at room temperature in Ringer's solution. The deflection occurring at each loading was determined and compared for each specimen.

The average deflection was .66 mm for the initial test compared to .63 mm for the group after boiling and cooling. The differences in individual specimens were of borderline significance ($t_9 = 1.455$). The mean deflection increased to .70 mm after 24 hours in Ringer's solution, but the change, compared to the values of the control group, was of borderline significance ($t_9 = 1.788$).

Deformation after one hour of dry heat

The third study utilized another group from Series Ten. Test conditions were similar to those in the previous experiment except that the specimens were heated in an incubator for one hour at 105° C.

After evaluation, the results observed were similar, but more pronounced and consistent than those in the boiling experiment. The average deflections were .69 mm in the control test, .65 mm after heating and immediate rewetting, and .75 mm twenty-four hours later. These differences were significant (t_9 immediately after rewetting was 287; t_9 the next day=2.245).

Several properties after four days of dry heat

The material in this experiment consisted of twelve specimens from subjects 11, 25, and 43. They were "sandwiched" and placed in an incubator for four days at 105° C. After removal, they were center-loaded, end-supported and tested to failure. Other conditions of testing were as on page 24. Ultimate fiber stress, modulus of elasticity, energy absorbed to failure, and total deflection to failure were calculated for each specimen. The group data were compared to wet controls from the air drying experiment on page 24. The data are summarized below in Table 7.

TABLE 7. *Changes in Physical Properties due to Heat Drying*

Hydration State	Number Samples	Ultimate fiber stress Kp/mm²	Modulus of Elasticity Kp/mm² × 10³	Energy absorbed to Failure Kp cm/mm² × 10⁻²	Total Deflection mm
Wet	23	18.2 ± 4.3	1.64 ± .37	9.14 ± 3.51	2.03 ± .45
Oven dried	12	17.2 ± 3.1	2.58 ± .37	0.157 ± .044	.54 ± .06
Probability		> .05	< .001	< .001	< .0005

It can be seen that heat drying at 105° C for four days significantly affected the modulus of elasticity, energy absorbed to failure and total deflection to failure parameters, but it did not significantly alter the breaking strength. These results in bending tests are in essential agreement with those of *Evans* and *Lebow* (1951) for tension tests. In addition, the load-deflection diagrams for the dry specimens were almost linear to failure, whereas wet specimens showed a major change at 40–50% of the breaking load. (Fig. 10)

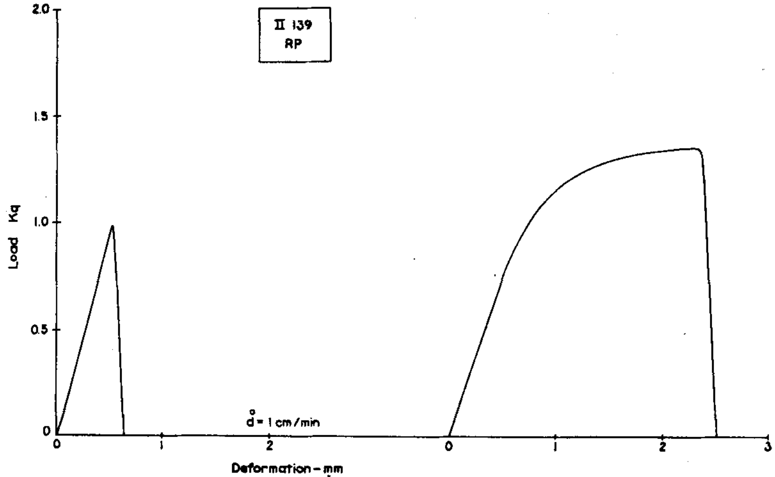


Figure 10. Bending load-deformation diagrams to show the effects of 4-day heat drying. At the left is the diagram for a specimen which had been heated to 105° C for four days. At the right, is its site-matched counterpart tested in the wet, fresh, state. The rate of deformation is indicated by $\mathring{d}$.

The four experiments as a group indicated that bone is permanently altered when subjected to 100—105° C for one hour. It appears that elevated temperature and dehydration produce a more pronounced effect than heating or drying alone. The effect of heat-drying for days and subsequent rewetting in the first experiment was to decrease the modulus of elasticity by 32% and, as was shown on page 27, the effect of drying at room temperature for the same time and rewetting was to decrease E by 5%. This feature was supported by the one hour tests in which it was shown that boiled specimens had a deflection increase of 6.1% twenty-four hours later and oven-heated specimens registered an 8.7% increase. It is felt that the effects of temperature elevation may vary with the length of exposure to heat, but the point at which the permanent changes begin has not been determined.

Stress-relaxation and hysteresis after four days of dry heat

Ten $2 \times 2 \times 50$ mm specimens from subject 33 were tested as follows: (1) wet in room temperature Ringer's solution; (2) dry, after incubation for four days at 105° C; (3) wet in Ringer's solution twenty-four hours later. The specimens, 30 mm in length, were arranged as cantilevers. The hysteresis test was performed by loading the specimens to 250 grams and then unloading from this point at rates of 0.5 cm/minute. After two minutes, without changing the position of the specimen, the stress relaxation test was conducted. In these, the specimen was loaded to 250 grams, the excursion of the load heads stopped and the recorder allowed to run for one minute. In effect, the deformation in a specimen was held constant while the recorder registered the change in resistance of the specimen over time. These operations were repeated on every specimen for the three conditions.

In each test, it was possible to measure the decrease in stress in each specimen after one minute and the residual deformation in the cycle testing when the specimen registered no resistance. The changes in area within the loops were obvious. (Fig. 11)

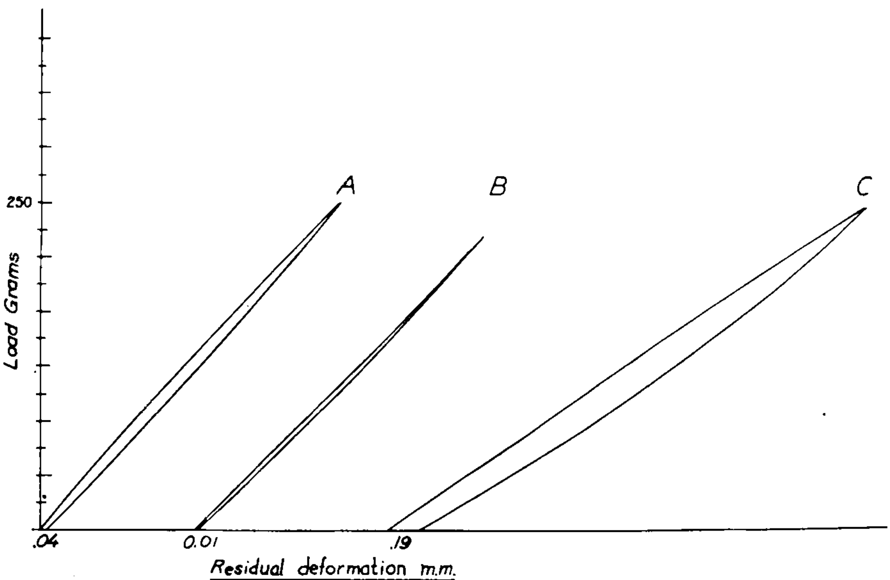


Figure 11. Hysteresis loops of one sample. Left, control test in Ringer's; Center, after five days at 105° C; Right, replacement in Ringer's. The residual deformation at zero load is indicated on the abscissa. See text for experimental method.

In the hysteresis tests, the average residual deformations at zero stress were 0.05 mm for the control group, 0.01 mm for the heat dried group and 0.21 mm for the rewet group. These changes were highly significant. The differences in the hysteresis loops for the three conditions are obvious in Figure 11. In the stress relaxation tests, the average decreases in stress after one minute were 4.3% in the control test, 2.1% in the incubated group and 13.4% in the rewet group. Again, the changes were highly significant.

Measurement of heat

The above demonstration of the effects of heat on physical properties indicated that temperature changes during the specimen preparation process should be investigated. When, *Rafel* (1962) measured the heat produced in mandibles by dental burrs, he observed that the longer the application of frictional stimulus the more widespread the heat effect and the longer the dissipation period. The highest temperature he measured some 3 mm from the cutting surface was 24° C. *Thompson* (1958) found that bone temperature could rise from 38°—67° C as the speed of drilling increased.

In this study, the only processes which might have raised the temperature of specimens above 37° C were machine grinding and milling. Since the milling blades always felt cool immediately after use due to cooling with 96% alcohol, it was felt that the bone surface temperature remained below 37° C during the process. As a similar statement could not be made regarding the surface temperature after machine grinding under water, the following experiment was performed.

Bone temperature during underwater grinding

Into five compression blocks from subjects 18 and 21, three Honeywell thermocouples 0.1 mm in diameter were introduced through holes which had been drilled just large enough to accommodate the lead and insulating jacket. The thermocouples were placed at varying distances from the potential grind surface (Figure 12). The specimens were mounted in a vise and put in a pan filled with room temperature water (23° C). A fourth thermocouple was placed in the water The four thermocouples were connected to an amplifier which, in

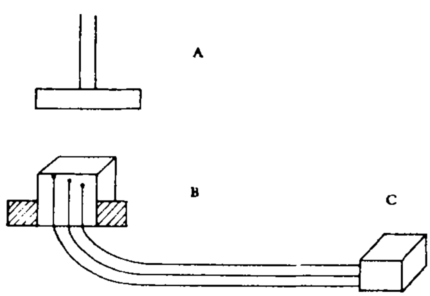


Figure 12. Diagram of apparatus used in measuring of heat during grinding. A. Grindstone which was lowered at a constant rate. B. Bone specimen with the thermocouples inserted. C. Amplifier.

turn, was linked to a four-channel recorder. This permitted simultaneous reading of temperature changes at three points within the specimens and in the surrounding fluid. Grinding was performed with a carborundum wheel which was put in contact with the bone surface and then lowered at a constant rate.

The set-up was calibrated for 40° C and 100° C by placing a steel die, which had been heated to the appropriate temperature, on the surface to be ground and recording the changes. The conditions of the tests conducted were:

1. Intermittent grinding with approximately ten seconds between operations in increments of 0.1 mm at 2800 rpm; grinding carried out parallel to the long axis of osteons and continued until the thermocouples were destroyed.

2. Constant grinding, parallel to the long axis of the osteons, with a wheel rate of 2800 rpm and bone grinding rate of .02 mm/sec; grinding continued until all thermocouples were destroyed.

3. Constant grinding of two specimens perpendicular to the long axis of the osteons; other conditions as in 2.

4. Constant grinding parallel to the long axis of the osteons at a wheel rate of 1000 rpm; bone grinding rate, 0.02 mm/sec.

The results were as follows: With a surface temperature of 100° C, the calibration curves showed that the specimen temperature rose to 83° C at a distance of 0.4 mm from the surface, to 65° C at 1.3 mm from the surface, and to 55° C at 2.7 mm from the surface (Figure 13).

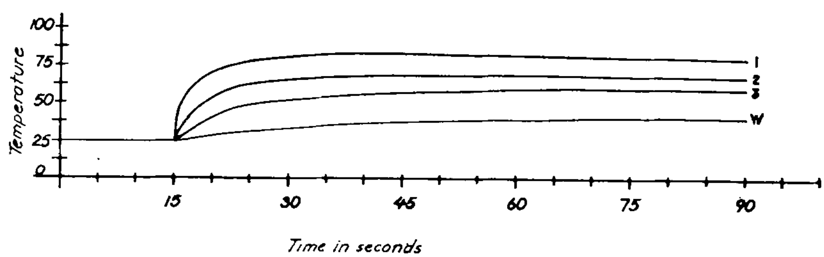


Figure 13. Calibration curves at 100° C. A steel die at 100° C is placed on the bone surface 15 seconds after the recording has begun. The three thermoelements within the bone sample, 1, 2, 3, then registered temperature rises as indicated, as did the element in water W. The distance of the thermoelements from the surface is as in the text.

With a temperature of 40° C at the surface, these temperatures were 36° C, 32.5° C, and 29° C, respectively. As would be expected, the time at which each element registered its maximum rise varied directly with the distance from the heat source. The grinding procedures demonstrated that the surface temperature did *not* rise above 40° C irrespective of the grinding plane. In the intermittent grinding test, there was no temperature increase until the respective thermoelements were approximately 0.1 mm from the grinding surface. During the constant grinding tests, the temperature in all elements slowly rose as they approached the grindstone (Figure 14). The results support *Rafel's* observation that, with intermittent grinding and adequate water cooling, bone temperature tends to remain at the level of the surrounding fluids. All grinding tests proved that the process utilized in specimen preparation did not produce an elevation greater than 40° C anywhere in the specimen.

The effect of testing at different temperatures

Smith and *Walmsley* (1959) in reporting observations on two samples of bone stated that the value for Young's modulus was inversely proportional to the temperature and that the effects of temperature variation in the range tested (40°—110° F) were fully reversible. Other information relative to the effect of this variable on bone testing is lacking. Due to its obvious importance, the following tests were performed.

Several properties at two temperatures

Seventy-seven specimens approximately $1.5 \times 1.0 \times 25$ mm from subjects 9, 34 and 39 were used. They were divided into two groups, care being taken to site-match samples in both groups. One group was tested in Ringer's solution at 21° C and the other group tested at 37° C. All specimens were tested to failure at a rate of deformation of 1.0 cm/min. with distance between supports of 20 mm. The ultimate fiber stress, modulus of elasticity, total deflection to failure and energy absorbed to failure were calculated for both groups.

The data as summarized in Table 8, indicated little difference in the values obtained for ultimate fiber stress, modulus of elasticity and energy absorbed to failure whether testing at 21° C or 37° C. There was, however, a significant increase in the total deflection to failure at 37° C. It is possible that a small change in a property might be masked by variation in individual specimens. This possibility was controlled in the next experiment.

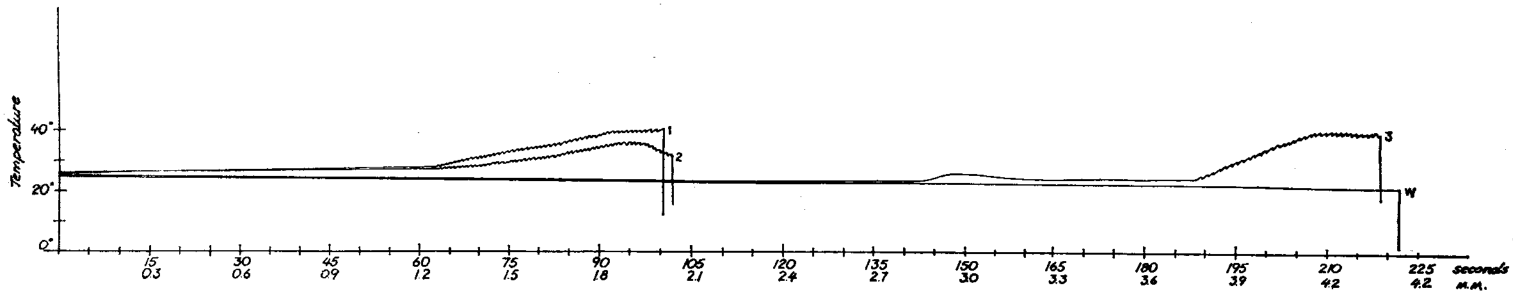


Figure 14. One set of curves obtained during the grinding process with set-up as in Figure 12. Grinding begins at the left side of diagram and continues until the 3 thermoelements are destroyed as indicated by the sudden drop below the base line.

TABLE 8. *The Effect of Temperature Upon Some Bending Properties of Cortical Bone*[1]

Temperature	No. of samples	Ultimate Fiber Stress-Kp/mm²	Modulus of Elasticity Kp/mm² × 10³	Total deflection to Failure mm	Energy Absorbed to Failure Kpcm/mm³
21° C	41	20.1±2.5	1.72±.23	1.47± .38	.121± .037
37° C	36	20.6±2.9	1.72±.25	1.69± .53	.126± .040
Probability		>.05	>.05	$.05>p>.01$	>.05

[1] All values are mean scores presented with one standard deviation.

Four curves were discarded for calculation of values other than ultimate fiber stress, thus there are 39 and 34 values respectively in the other groups.

Deformation at two temperatures

First, one group of specimens from Series Ten was placed in Ringer's solution at 27° C and tested; then they were warmed for one hour in 37° C Ringer's solution and retested. The specimens were tested as cantilevers with a length of 30 mm up to a load of 100 grams at a rate of 1.0 cm/min. Deformation occurring in both instances was determined and the differences in individual specimens analyzed.

The specimens had an average deformation of 0.77 mm at 21° C and 0.82 mm at 37° C. The moduli of elasticity changed accordingly from 1250 Kp/mm² at 21° C to 1190 Kp/mm² at 37° C. These differences were significant (t_9 differences=3.028).

The observations in this experiment thus confirm *Smith* and *Walmsley's* finding of a decrease in the modulus of elasticity associated with an elevation in temperature. It is reasonable to conclude on the basis of the observations in this pair of experiments that deformation in bending samples is greater at 37° C than at 21° C. The magnitude of this change, 6%, might be small enough to be obscured in those tests in which identical specimens were not used. Thus, there might be an effect of temperature upon the breaking strength, but it would require an inordinate number of subjects and samples for demonstration.

The effect of repetitive loading

Clinically, it has long been known that bones fatigue and fail with constant repetitive loading. *Smith* and *Walmsley* (1957) stated that repeat stress well below the ultimate strength ultimately causes a fatigue fracture and that the number of stress applications necessary is an inverse function of the magnitude of the stress. *Evans* and *Lebow* (1957) found the average fatigue life of 15 femoral samples tested in bending to be approximately 1.2 million cycles under a constant load of approximately 3.5 Kg/mm^2. *Lease* and *Evans* (1959) performed fatigue studies on intact human metatarsals and demonstrated fatigue failures from 1,000—14,000 cycles with 15 lb. loads. Since repetitive loading was employed in the current experiments to determine the effects of variables upon the physical properties, a question naturally arises concerning its possible influence on the results. This problem was checked by the following formal tests and by observations in the course of other tests.

Repetitive loading of cantilever beams

Three groups from Series Ten were repetitively loaded ten times as follows: one group was loaded to 100 grams with thirty seconds between loads; the second group was loaded to 100 grams with 3.5 seconds between loads; the third group was loaded to 200 grams with thirty seconds between loads. Tests were performed in Ringer's solution at room temperature with a deformation rate of 1 cm/min and beam length of 30 mm. The specimen positions were not changed between loadings. The average stress values were 3 Kp/mm^2 in the 100 gram tests and 6 Kp/mm^2 in the 200 gram tests.

The deformation resulting from the loads above was determined for the 300 tests. The average deflection from the applied load in the first group for the first loading was .69 mm and, at the tenth loading, .68 mm. This decrease was not significant. In the second group, the average deflections were .64 mm in both instances. In the third group average deflection for the first loading was 1.25 mm and for the fifth and tenth times, 1.23 mm. No significant changes were noted before the fifth load. This 1.7% decrease in deflection occurring at the fifth load was significant (t_s differences—5.013).

Repetitive loading of centrally-loaded, end-supported beams

Eleven end-supported bending specimens from subject 29 were loaded repetitively four times to 500 grams at a rate of 1 cm/minute. Specimens were approximately 1.75×1.75 mm in cross-section dimensions and were tested over a support length of 27 mm. This load produced an average maximum fiber stress of 3.83 Kp/mm^2. The average deformation was calculated in each case and was found to be almost identical in the first and fourth loads in all specimens.

From the two experiments, it can be seen, that bending specimens can be reloaded without any effect if a small unit stress is maintained. It is felt that there is some limiting unit stress which will result in permanent deformation with repetitive loading. This limit is not defined here since it is expected that it will vary from bone to bone and subject to subject (*Sedlin* and *Hirsch*, 1965.) These data do not mean that repetitive loading can be carried out indefinitely without effect even with the small loads utilized, for these tests have not set an upper limit regarding the number of times a specimen could safely be reloaded. It is concluded, with unit fiber stresses in the 3—4 Kp/mm² range, that specimens can be safely reloaded at least ten times when set-up as cantilevers and four times when set-up as end-supported, centrally loaded beams.

The influences of site of sampling, age and microanatomy

Site of sampling

Physical properties have been repeatedly demonstrated to differ from individual to individual, bone to bone and in different parts of the same bone in many investigations utilizing a variety of techniques (*Carothers et al*, 1949; *Calabrisi* and *Smith*, 1951; *Evans* and *Lebow*, 1951; *Evans*, 1957; *Yamada* and *Motoshima*, 1960; *Evans*, 1964; *Hirsch* and *Evans*, 1965). It is apparent that physical properties of bone cannot be discussed in terms of the actual values without citing the location of the source of specimens. Samples which had been tested wet as end-supported, centrally loaded beams with the same constant rate of deformation were analyzed with respect to site within the mid-femoral diaphysis. In Tables 9 and 10, it can be seen that there are differences in physical properties which can be attributed to source within the mid-femur. On the basis of the data, it can be stated that the posterior quadrant samples are weaker and have a lower modulus of elasticity than samples from other quadrants which do not differ significantly from each other. The findings here relative to weakness of posterior quadrant samples are in agreement with those of *Maj* (1938). In five of the subjects, specimens from both femurs were tested. Analysis of these values failed to demonstrate significant differences in right and left femur although the numbers of specimens available for the comparison were small.

As would be expected from the data in Tables 9 and 10, there is a high correlation between ultimate fiber stress and the modulus of elasticity in bending samples from the mid-femur. The data, presented elsewhere (*Sedlin* and *Hirsch*, 1965), showed a product-moment correlation coefficient of +.81,

TABLE 9. *Changes in Ultimate Fiber Stress in Bending Samples Grouped According to Location Within the Mid-Femoral Diaphysis*[1]

Subject	Quadrant of Origin				Mean
	Posterior	Lateral	Anterior	Medial	
9	18.7	19.4	20.8	18.3	19.3
11	19.5	23.1	23.8	19.5	21.5
14	20.9	21.9	22.9	22.0	21.9
15	17.2	17.3	17.9	16.4	17.2
23	18.3	20.6	18.3	20.1	19.1
25	17.7	20.5	19.8	19.8	19.5
28	19.8	20.4	19.0	19.6	19.7
34	20.7	23.0	18.0	20.4	20.5
36	16.6	18.2	17.6	17.9	17.6
38	18.1	18.3	19.7	19.1	18.8
39	20.7	20.8	21.5	22.3	21.3
41	9.6	11.5	11.8	12.4	11.3
43	14.0	15.9	15.1	15.6	15.2
Mean	17.8	19.3	18.9	18.7	18.7

Analysis of Variance

Source of Variation	Degrees Freedom	Sum of Squares	Mean Square	F	Probability
Total	51	460.73	9.03		
Subjects	12	404.25	33.69	29.30	< .01
Location	3	15.25	5.08	4.41	< .01
Residual	36	41.23	1.15		

[1] All values are reported in Kp/mm^2 and here are presented as mean values for all samples from a quadrant for a subject. 336 individual tests are represented in the table.

which was highly significant, and the following estimating equations were derived:

$$\hat{\sigma}=4.0+9.45E$$

$$\hat{E}= .264+ .06975\sigma$$

where $\hat{\sigma}$ and $\hat{E}=$ estimated stress and modulus of elasticity respectively.

These findings are in agreement with those of *Maj* (1942) in his study of human bending samples, *Toaiari's* (1937) study of mule metatarsal bending samples and *Weir et al,* (1949) in a study of intact rat femora.

TABLE 10. *Changes in Modulus of Elasticity in Bending Samples Grouped According to Location Within the Mid-Femoral Diaphysis*[1]

Subject	Quadrant of Origin				Mean
	Posterior	Lateral	Anterior	Medial	
9	6.53	6.25	7.38	5.96	6.53
11	6.83	7.80	7.55	6.39	7.14
14	6.60	6.47	7.98	7.72	7.19
15	5.69	5.55	5.53	5.20	5.49
23	5.14	5.52	5.62	5.77	5.51
25	6.31	7.10	6.99	6.73	6.78
28	6.44	6.92	6.09	7.15	6.65
34	6.71	7.52	6.05	7.04	6.83
36	5.47	5.83	5.49	6.16	5.74
38	6.54	6.75	6.76	7.24	6.82
39	6.86	6.28	7.12	6.60	6.72
41	3.56	4.34	3.99	4.55	4.11
43	5.38	5.71	5.25	4.94	5.32
Mean	6.00	6.31	6.29	6.27	6.22

Analysis of Variance

Source of Variation	Degrees Freedom	Sum of Squares	Mean Square	F	Probability
Total	207	15.3393	0.0741		
Subjects	12	9.2803	0.7734	25.36	$< .01$
Location	3	0.2012	0.0671	2.20	$\cong .10$
Residual	192	5.8578	0.0305		

[1] Values are $Kp/mm^2 \times 1000$. This analysis was carried out utilizing four values for each quadrant for each subject with occasional insufficient values being substituted by randomization. Each value reported should be multiplied by .25 for a mean value for a quadrant or subject.

Age

Maj (1942), when studying bending strength of central diaphysial samples of multiple human bones, felt that there was a parabolic relationship between age and strength in adults with low points in the 25—30 and 60—80 age groups; but there was considerable variation in the data presented. *Aochi et al* (1959) demonstrated a decrease in the maximum transverse compressive load of the middle of the shaft of long bones with increasing adult age. *McElhaney* (1964) stated that the ultimate strength of compact bone decreased after age 26. Yet, *Evans* and *Lebow* (1951) were unable to detect differences in the tensile and shearing strength of adult femoral samples with advancing age. A similar finding was reported by *Ascenzi* and *Bonucci* (1964) regarding the strength of single osteons. Other investigations (*Hollinghaus*, 1959 and *Vose et al*, 1961) have tended to indicate that the intrinsic strength of bone samples increases with advancing age.

Analysis of some data from this study presented elsewhere (*Sedlin* and *Hirsch*, 1965) and in Tables 9—12, demonstrated no trend with respect to age. The inconclusiveness of these findings, coupled with the disagreement noted in the literature, does not mean that physiologic age is not a determinant of physical properties. Rather, a systematic study in which other variates are standardized needs to be done before valid conclusions can be drawn.

Microanatomy

Maj (1938), in studying bending samples of bull bone, reported no correlation between porosity of samples and bending strength when the porosity (Haversian canal volume) was less than 50%. *Evans* (1958) felt that a sample with a few large osteons and irregular fragments would have a greater tensile strength than a sample with a larger number of small osteons and fragments. This statement, while valid as a generalization accounting for the differences between fibula and femur, did not account for differences in samples from the same bone. *Currey* (1959) stated that bone without Haversian systems had a greater tensile strength than bone with Haversian systems and calculated a strong negative correlation ($r = -0.6629$) between tensile strength and percentage of "Haversian system" type bone present. *Vose* (1962) found no correlation between ultimate yield loading and the size and frequency of Haversian canals.

The question of the effect of porosity was studied by preparing sections from 125 specimens from four subjects that had been tested to failure under identical conditions. At least four cross sections from each specimen were prepared by *Frost's* techniques (1958, 1959). Specimens were selected for study on the basis of disparity in breaking strength, so that approximately equal numbers of specimens with high breaking strength and low breaking strength were studied. The average Haversian canal volume of each specimen was determined by previously reported methods (*Hennig*, 1958; *Sedlin et al*, 1963; *Sedlin* and *Hirsch*, 1965), and this value was expressed as a percentage of the volume of the specimen.

It was found that there was no correlation between the Haversian canal volume and individual values for ultimate fiber stress and energy absorbed to failure in the majority of the specimens examined. At the extreme values of porosity — less than 2.5% or greater than 15% — an inverse relationship was noted with the breaking strength; but this relationship, involving less than 20% of the specimens, was far from absolute. There was a weak correlation between porosity and the modulus of elasticity but it was not significant. These findings substantiate those of previous authors indicating little relationship between Haversian canal volume and physical properties. Yet, it cannot be concluded that the porosity of bone samples is not an important determinant of the physical properties. The problem is too complex to be accounted for solely by determination of the porosity volume (*Sedlin* and *Hirsch*, 1965).

Olivo et al (1937) found that bending specimens which had a higher proportion of vertical or steeply oriented collagen fibrils had a higher modulus of elasticity and a higher resistance to fracture than specimens which had circular or obliquely oriented fibrils. *Toaiari* (1937) stated that resistance to fracture was greater where the collagen fibrils were parallel to the axis of the test section than when they diverged. *Evans* (1958) agreed with these findings and suggested that the greater the number of cement lines in a specimen, the weaker the bone will be. According to *Ascenzi* and *Bonucci* (1964), the ultimate tensile strength in single osteons with a marked longitudinal arrangement of fiber-bundles in successive lamellae is higher than those with fiber-bundles running alternately so that their direction in successive lamellae changes through an angle of about 90°. Thus, there appears to be agreement, insofar as bending and tension are concerned, on the fact that a general longitudinal orientation of fiber-bundles provides greater strength than divergent or oblique orientation.

Consideration of the difficulties of *Ascenzi* and *Bonucci* (1964) in locating osteons with a longitudinal course discloses another feature to be noted. In adult bone, osteons have straight vertical paths for short distances, if at all. Constant branching and overlapping produces an irregular pattern in cross-section. Therefore, in a tension test, some osteons will be pulled in their long axis, while others will be subjected to shear, compression, or bending. A reasonable conclusion, therefore, is that a study of the relationship between microanatomy and physical properties in small samples presents a complex problem in three-dimensional reconstruction.

Interim summary

1. Freezing of samples of cortical bone has no effect on physical properties when tests to failure are done.

2. Preservation of samples in formalin may produce an increase in the modulus of elasticity in tension testing.

3. Preservation of samples in alcohol decreases the deformation of bending samples, an effect that may or may not be reversible.

4. Air drying of samples has the following effects in bending:
   a. Increase the ultimate fiber stress after one hour of drying.
   b. Increase the modulus of elasticity after five days of drying.
   c. Decrease or no change in the modulus of elasticity after one hour of drying.
   d. Decrease in the modulus of elasticity after rewetting.

5. Heating of bending samples to 100—105° C has the following effects:
   a. Decrease the ultimate stress, fiber but not significantly in specimens tested dry.
   b. Significantly increase the modulus of elasticity, decrease the energy absorption to failure, and the total deflection to failure in specimens tested dry.
   c. Significantly decrease the modulus of elasticity in specimens tested after rewetting.
   d. Decrease stress-relaxation, hysteresis and residual deformation after cyclic loading in specimens tested dry.
   e. Increase the parameters above (d) after heat drying and rewetting.

6. Drying at room temperature or 105° C produces significant changes in the size of small samples.

7. The permanent effects of temperature elevation and drying combined are greater than either alone.

8. Cortical bone transmits heat quickly at least as far as 3 mm from the heat source.

9. Temperature does not rise above 40° C when bone is ground under water.

10. The deformation of bending samples is significantly greater at 37° C than at 21° C. No changes are noted in the ultimate fiber stress and the energy absorbed to failure.

11. Bending samples can be loaded repetitively at least ten times as cantilevers and four times as end-supported beams if unit-stresses are maintained in the 3—4 Kp/mm^2 range.

12. The posterior quadrant of the mid-femur has a lower ultimate fiber stress and modulus of elasticity than other quadrants.

13. A high correlation exists between the ultimate fiber stress and the modulus of elasticity in bending samples.

14. A definite association between physical properties and chronologic age is not seen.

15. Little or no correlation between the Haversian canal volume and physical properties has been observed in 125 samples.

CHAPTER IV

DERIVATION OF A RHEOLOGIC MODEL FOR BONE

Introduction

In the broad sense, rheology is that branch of physics which is concerned with the deformation and flow of materials. It deals with all materials, save the theoretically ideal liquid and absolutely rigid solid. The goal of rheology is to predict the forces necessary to cause a given body to flow or deform, or prediction of the flow or deformation that will result from a known combination of forces. The concepts to be presented in this section are relatively old in the realm of theoretical mechanics. The author has utilized standard works in the rheological literature for definitions and organization. (*Jaeger*, 1956; *Reiner*, 1958; *Reiner*, 1960; *Fredrickson*, 1964). The following represents an attempt to place the study of physical properties of bone into the theoretical frame of reference of rheology.

Fundamental rheologic properties

There are four fundamental rheologic properties of materials — (1) elasticity, (2) viscosity, (3) plasticity, (4) strength. The majority of more complex material properties can be described by combining the fundamental properties in various ways. Fundamental properties and the complex properties that can be derived from them are termed *essential properties* (*Reiner*, 1958). First, consider the fundamental properties, bearing in mind that *all real materials possess each property, although quantitatively the amount of each property may be negligible.*

Elasticity

Elasticity is defined as that property of a material which enables it to recover from a deformation produced in the material by an external force. In a *perfectly elastic solid*, all deformation is recovered upon release of the external forces and strain is proportional to stress for all stresses. A suitable model for the perfectly elastic solid is a spring as in Figure 15. A stress-strain diagram will be a straight line (Figure 15) and the following relation between stress (σ) and strain (ϵ) is present.

$$\sigma = E\epsilon \tag{1}$$

where E is an elasticity constant for the material.

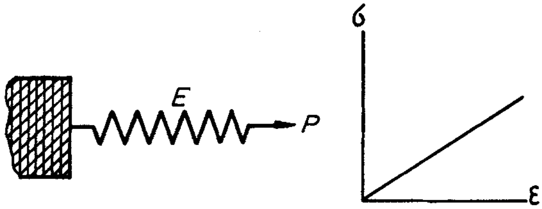


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Figure 15. The Perfectly Elastic Solid or Hooke Body. **A.** It is symbolized by a spring which has a linear relation between pull or load (P) and deformation. **B.** A stress-strain diagram for this body is a straight line. Notation is in the text. The strain produced by an instantaneous stress is also attained instantaneously and upon cessation of the stress, the strain recovers instantaneously.

Upon application of a load, the resultant deformation is reached instantaneously and, upon release of the load, original shape is regained. The perfectly elastic solid is also called the Hookean body.

Viscosity

Viscosity is the property of a substance which determines its resistance to shear stress or its ability to flow. It is a measure of the lack of slipperiness of the material, and, a material possessing viscous properties will flow under the action of any shear forces however small. In the *perfectly viscous fluid* or Newtonian substance, stress is proportional to the rate of strain ($d\epsilon/dt$). The Newtonian body can be symbolized by a dashpot; a tube containing some viscous substance in which a stopper is loosely fitted (Figure 16). The relationship between stress and the rate of strain is a straight line as expressed by

$$\sigma = \eta \frac{d\epsilon}{dt} \tag{2}$$

where η is a viscosity constant for the material.

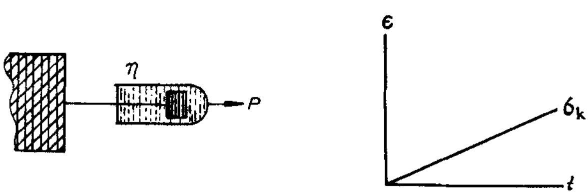

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Figure 16. The Perfectly Viscous Substance or Newton Body. **A.** The model is a test-tube containing some liquid in which a plunger is fitted — a dash-pot. If a force (P) is applied, the tube has a variable resistance to movement on the plunger, dependent, in part, upon the coefficient of viscosity of the liquid (η). **B.** A strain-time diagram with stress constant (σ_k) is shown.

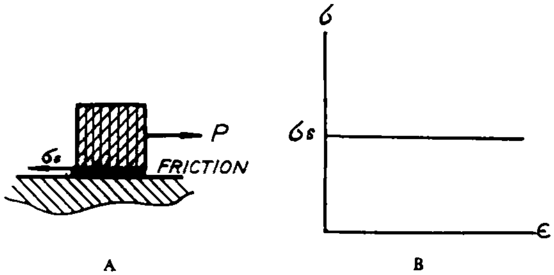

Figure 17. The Rigid Plastic Body or St. Venant Substance. **A.** This is symbolized by a weight resting on a solid surface which resists movement by its frictional contact with the solid surface which is indicated by σ_s. The object does not move until this resistance has been overcome. **B.** A stress-strain diagram will show for all stresses below σ_s no deformation and for stresses above σ_s an indeterminate deformation.

Non-Newtonian or non-linear viscosity exists in many substances. Here, stress is some function of the strain rate, but not proportional to it. Although of great importance, since blood and interstitial fluid are non-Newtonian, (*Bayliss*, 1952) further discussion of this type of behavior is beyond the scope of this study.

Plasticity

Plasticity is property of a material which enables it to resist deformation until a certain force has been applied, subsequent to which it deforms indefinitely. The point at which the internal friction which resists deformation, is overcome is known as the *yield point*. The *rigid plastic*, St. Venant body, can be symbolized by a weight resting on a tabletop with a solid friction between them (Figure 17). Below the yield point (σ_s), no deformation whatsoever occurs. When the stress becomes equal to the yield stress, it flows indefinitely (Figure 17). So,

$$\sigma < \sigma_s; \quad \epsilon = 0 \tag{3}$$

$$\sigma = \sigma_s; \quad \epsilon \text{ indeterminate}$$

The *perfectly plastic solid* or *Prandtl body* exhibits elastic behavior below the yield point and then flows indefinitely at the yield point. It is symbolized by linking a spring and frictional contact in series (Figure 18) and has a stress-strain diagram as shown in Figure 18. Its relationships are

$$\sigma < \sigma_s; \quad \epsilon = \frac{\sigma}{E}$$

$$\sigma = \sigma_s; \quad \epsilon \text{ indeterminate}$$

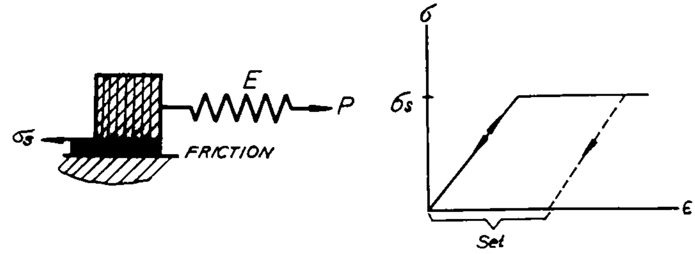

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Figure 18. The Perfectly Plastic Solid or Prandtl Substance. **A.** This is symbolized by a Hooke spring linked in series to the St. Venant friction element. This substance will have elastic deformation below the yield point of the plastic and above this will have an indeterminate plastic flow. This is pictured in **B:** if the stress is discontinued at some time, the elastic part of the material will then recover, however there will be permanent deformation. This is indicated by the *set* noted on the abscissa.

Complex Properties

The idealized bodies just cited rarely exist in nature in pure form — i.e. the Hookean, Newtonian, St. Venant and Prandtl substances are not to be found as isolated materials. By linking the elements, more complex behavior can be described. *If the elements are linked in series, each element then takes the same total load and the deformation is additive. If the elements are linked in parallel, each element takes a part of the load, while the deformation in each is identical.* The models already shown or to be described refer to elongations in tension, but can be modified to depict shear, linear and cubic dilatations. The equations that are presented refer to uniaxial tension, but they can be adapted to other stress conditions.

In consideration of complex properties the combination of a Hookean spring and Newtonian dash-pot in parallel and in series merits detailed examination, since these combinations form fundamental units of more complex models.

First, consider a spring linked in parallel to a dash-pot, Figure 19, and the properties represented by the:

Kelvin body or firmoviscosity

The term firmo-viscosity or solid viscosity is applied to that property exhibited by an elastic material which exhibits resistance to deformation in addition to elastic resistance. The model presented (Figure 19) is appro-

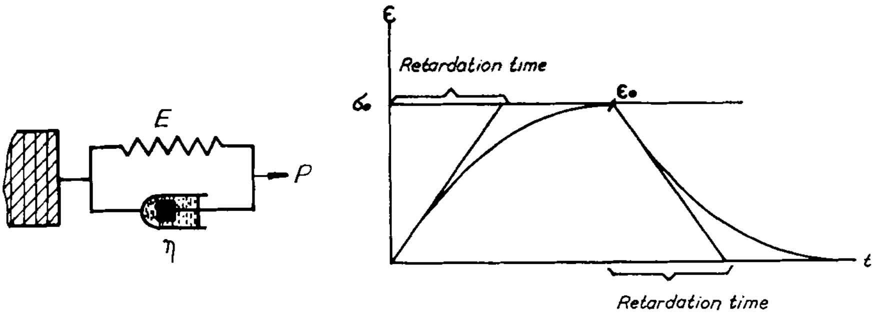

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Figure 19. The Firmo-Viscous Substance or Kelvin Body. **A.** A spring is linked in parallel to a viscous element. If a constant stress, σ_0, is applied, a strain-time diagram will be as in **B.** The deformation is not attained instantaneously but exponentially approaches some final deformation, the time required being dependent upon the constants for the spring and dashpot. The relationship of the constants for the dash-pot to the spring determines the amount of damping of instantaneous deformation which can be obtained. This is the retardation time. If at some certain deformation ϵ_0 the stress is discontinued, the body recovers its original length again with damping of instantaneous recovery as in the application of the load. This body eventually recovers all deformation.

priate for cork. Some aspects of the behavior of this model are as follows: From eq. (1) and eq. (2) the following relationship is evident

$$\sigma = E\epsilon + \eta \frac{d\epsilon}{dt} \tag{4}$$

If a constant stress (σ_0) is applied at time $t=0$ when $\epsilon=0$, the total strain will be given by

$$\epsilon = \frac{\sigma_0}{E}\left\{1 - e^{-\frac{Et}{\eta}}\right\} \tag{5}$$

Therefore, the strain σ_0/E which would be reached instantaneously in the case of a perfectly elastic substance is approached exponentially as in Figure 19. Thus, there is a delay in the attainment of the final deformation. Materials characterized by this model vary in the time required to reach total deformation. The *retardation time* is a material property defined as the time at which $1/e$ of the final deformation has yet to be achieved. This delay in attaining total strain under a constant stress is termed *elastic fore-effect.* Upon release of the load the strain decreases exponentially to zero according to

$$\epsilon = \epsilon_0 e^{-\frac{Et}{\eta}} \tag{6}$$

where ϵ_0 equals the strain at release of load. The time at which $1/e$ of the initial deformation is unrecovered is given by η/E. This delay in recovery is termed *elastic after-effect* and the combination of elastic fore-effect and elastic after-effect is termed *delayed elasticity*. To recapitulate, the Kelvin body is characterized by absence of instantaneous elasticity, an elastic fore-effect and elastic after-effect, no change in stress under a constant deformation, and ultimate recovery of all deformations.

Now consider a spring *linked* in *series* with a dash-pot and the properties described by:

The Maxwell body or elastico-viscosity

The model is shown in Figure 20 and is appropriate for pitch and concrete. Since the total strain (ϵ) is the sum of strain in both elements, and the same stress acts on both elements, eq. (1) and (2) yield

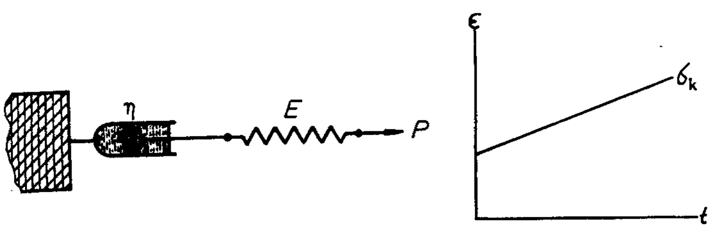
$$\frac{d\epsilon}{dt}=\frac{d\sigma/dt}{E}+\frac{\sigma}{\eta} \qquad (7)$$

If a constant stress (σ_0) is applied to this material when it is unstrained initially, the strain satisfies

$$\epsilon=\frac{\sigma_0}{E}+\frac{\sigma_0 t}{\eta} \qquad (8)$$

so that, there will be an initial elastic strain (σ_0/E) that is instantaneous, followed by an increasing viscous strain, which is linear, and a strain-time curve is as seen in Figure 20. The instantaneous elastic strain is recoverable, but the viscous deformation is not. This model will deform permanently with the smallest stresses. Under a constant strain (ϵ_0) there is a decrease in stress in the model according to

$$\sigma=E\epsilon_0 e^{-\frac{Et}{\eta}} \qquad (9)$$



A B

Figure 20. The Elastico-Viscous Substance or Maxwell Body. **A.** The model is a spring linked in series to a dash-pot. Relationships of this model are discussed in the text. It can be seen that with a constant stress the material will have initial instantaneous deformation due to the elastic element and secondary progessive deformation due to the viscous damper. A strain-time diagram will be as in **B.**

This phenomenon is called stress relaxation. The relaxation time is a constant for the material, and equal to the time in which stress decreases to $1/e$ of its original value. It can be seen from eq. 9 that stress under a constant strain will tend to zero.

Thus the Maxwell body and the property of elastico-viscosity can be characterized by instantaneous elasticity followed by progressively increasing deformation under constant stress, an exponential decrease in stress under constant deformation, and some permanent deformation after any stress.

The elements just described, fundamental and complex, can be combined to describe different behavior. By various combinations, most material properties can be depicted, thus providing a framework for a more complete analysis of the material's properties.

Experiments

The preceding chapter demonstrated that the physical properties of cortical bone could be altered by many factors. In the introduction to this chapter, some of the parameters by which a material is described were indicated. It can be seen that in order to characterize cortical bone, its behavior in a variety of circumstances must be determined. In this section, specific experiments that were necessary to further characterize cortical bone as a material are described.

The behavior of bone under a constant deformation

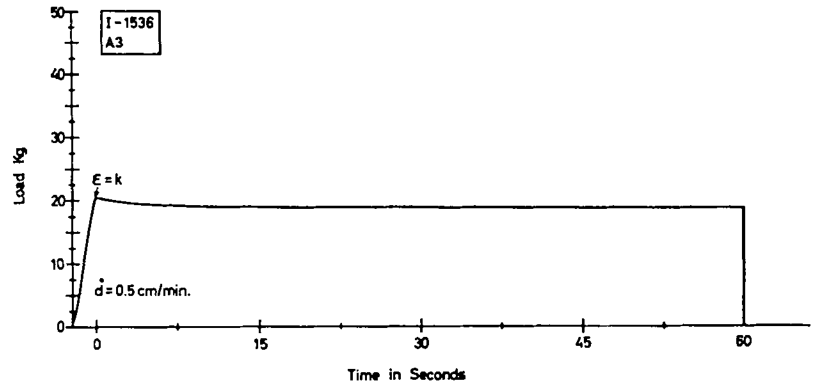
The test material included the following:

One group from Series Ten tested as cantilevers, 16 specimens from six subjects tested in tension; twenty-five specimens from 14 subjects tested as end-supported beams, five complete cross-section segments of femur from five subjects tested in longitudinal compression, and 15 compression blocks from the same subjects tested in compression.

The constant deformation was applied by loading the specimens at rates varying from 0.2 cm/min. to 1.0 cm/min., the rate being varied according to the size of specimen and type of load being applied. When the load reached an estimated 25% of the ultimate failure load, the progressive loading was stopped and the resultant deformation was maintained while the recorder was kept running. Thus, stress as a function of time with deformation held constant was recorded. The recording time was varied from test to test. Each test was watched, and the recording discontinued when it became evident that no further change was occurring. The predominant recording time was 30″ to 2′, but this was extended to 10 to 15′ periods in certain instances in order to confirm the fact that additional changes were not occurring.

The *qualitative* result was *identical* in all specimens from all subjects. When deformation became constant (i.e. loading stopped), a decrease in stress occurred with 50–60% of the decrease being apparent at 30″. Sample curves are shown in Figure 21. The curves all were asymptotic to some new level, but none tended to zero. There were quantitative differences in the amount of decrease in stress in time according to the axis of loading, but these differences are not germane to the issue since they undoubtedly furnish another example of the heterogeneity of bone.

This experimental group in addition to the experiments cited on page 30 thus demonstrate that *stress relaxation* is a phenomenon present in bone and that bone has a relaxation time as a material property. Stress relaxation



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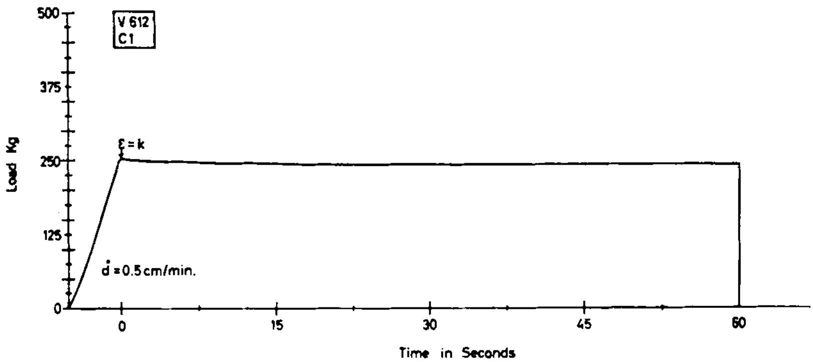

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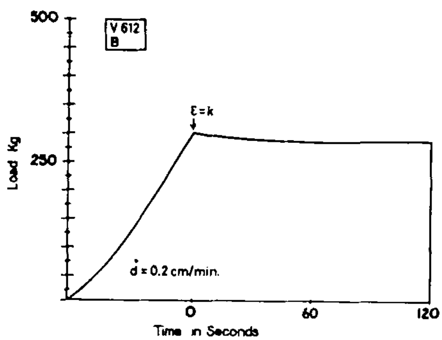

C

Figure 21. Stress relaxation in cortical bone. **A.** A tension test. The specimen is deformed at 0.5 cm/minute until a load of 20 Kg. is reached. The progressive loading is then stopped and the deformation held constant, indicated by $\epsilon = K$. The recorder then registers the decrease in stress during the next 60″. **B.** A test in compression on a $5 \times 10 \times 15$ mm block. **C.** A test on a 5 cm segment of central diaphysis.

has been noted in muscle (*Stacy et al*, 1955; *Schottelius*, 1957), joints (*Johns* and *Wright*, 1964), skin (*Ridge* and *Wright*, 1964) and was previously stated to occur in bone (*Smith* and *Walmsley*, 1957).

Changes in the deformation of bone at different rates of deformation up to a constant load

One group from Series Ten, twelve cantilever specimens from subject 35, and five tension specimens from five subjects were used in this experimental group. Cantilever specimens from Series Ten were loaded to 100 gms, first at a rate of 0.1 cm/min., then at rates of 1.0 cm/min. with two minutes being allowed between loads. They were then refrigerated over night and the tests repeated the following day, but this time they were loaded at a rate of 1.0 cm/min. at the first trial and 0.1 cm/min. on the second trial. Two minutes were allowed for recovery between trials and the specimen position was not changed between trials. Specimens from subject 35 being larger than the Series Ten group were loaded to 250 grams, first at a rate of 1.0 cm/min., and then at 0.1 cm/min. As with the Series Ten specimens, two minutes was permitted for recovery and the specimen position was not changed between loads. Tension specimens were loaded to 10 kg, first at a rate of 0.1 cm/min. and then 0.2 cm/min. More rapid loading rates were precluded due to the relatively slow speed of the recording as noted in Chapter II. Unit stress approximated 3.5 kp/mm^2 in the cantilever specimens and .8 kp/mm^2 in the tension tests. Deformation occurring at the different rates of loading was determined, and the differences analyzed.

In the Series Ten group, the average deformation was .86 mm in the first test at rate of 0.1 cm/min. and 0.82 mm at a rate of 1.0 cm/min. The following day, when the specimens were tested with the more rapid rate initially, mean deformation at 1.0 cm/min. was 0.85 mm and at 0.1 cm/min., 0.87 mm. The differences in individual tests were highly significant in both instances ($t_9=4.56$ first test; $t_9=3.56$ second test). In the 12 specimens from subject 35 ,mean deformation at the rate of 0.1 cm/min. was 1.36 mm and at the rate of 1.0 cm/min. the deformation was 1.27 mm. These results were consistent and highly significant ($t_{11}=6.90$). The results were similar in the tension group, i.e. mean deformation at 0.1 cm/min. was .11 mm, while at 0.2 cm/min., it was .09 mm. The individual differences were significant ($t_4=2.412$).

These results for cantilever bending and tension demonstrate that bone deforms less with rapid loading conditions than with slower conditions. *McElhaney* (1965) demonstrated similar behavior in testing compression samples of human and bovine bone at more rapid rates of loading than were used here. It is thus evident that deformation of bone is some function of the rate of deformation and that the modulus of elasticity of bone is, in reality, a range of values.

Behavior of bone during loading and unloading at different constant rates

Twenty-five cantilever specimens from 2 subjects, a group from Series Ten, five compression blocks from 3 subjects, 15 simple beam specimens from 2 subjects, and 10 tension specimens from 5 subjects were used to study this feature. The test machine was programmed to cycle between zero load and 50% of full scale, with appropriate small loads being chosen for each type of specimen (100 or 250 gms for a cantilever test and 100 kg for a compression block). Two tests were performed on each specimen, one at 0.1 cm/min. or 0.2 cm/min. and the second at 1.0 cm/min.

The general shape of the hysteresis loops differed according to the axis of loading, but in all tests a loop was described and the findings were similar. An example of one pair of loops is seen in Figure 22. The steeper, more nearly linear curve at the more rapid rate of loading is to be expected from the preceding experiment. It was shown in the previous chapter that the residual deformation could be affected by previous heating of bone. The

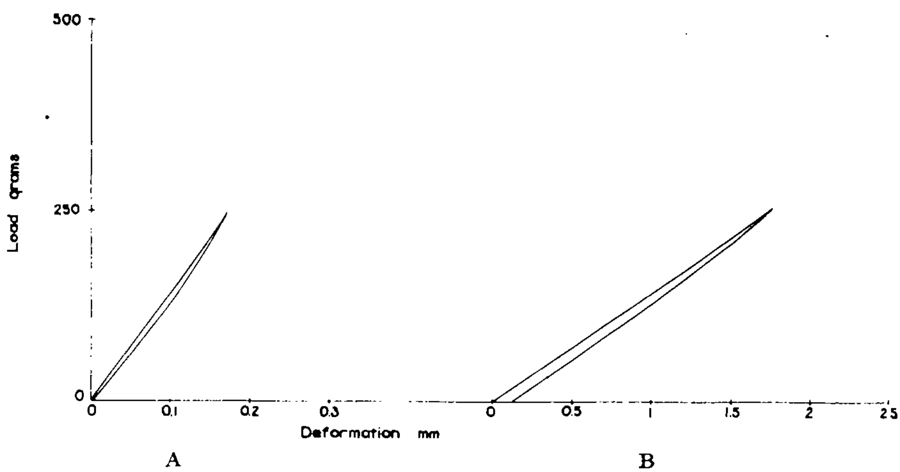

Figure 22. The effect of different deformation rates upon hysteresis in bending samples. **A.** The specimen was loaded at a rate of 1.0 cm/minute up to a load of 250 grams and then unloaded from this point. **B.** The same specimen at a rate of 0.1 cm/minute. Note the decrease in residual deformation at zero load in test **A.**

new feature that was demonstrated by these tests was the fact that the amount of residual deformation varied with the rate of deformation. More residual deformation was present at zero stress at the slower rate, than at the more rapid rate. These hysteresis loops thus indicate that more energy is dissipated in this type of testing with a slow rate as compared to a more rapid rate.

The characteristics of bone deformation under a constant load — creep studies

Part 1

This was a preliminary investigation, using 4 tension samples from subject 37 tested on the Instron machine with the machine programmed to deliver a constant load $\pm 1\%$. Each specimen was tested first at 5 kg and then at 10 kg. These loads represented approximate stresses of 0.85 and 1.7 kp/mm^2, respectively. The constant load was maintained for 30′ and readings of deformation were made at 1′, 2′, 3′, 4′, 5′, 15′, and 30′. The load was released at 30′, and 30′ were allowed for recovery before the second load was applied.

It was seen with the 5 kg loads, that the final resultant deformation was attained almost immediately in each specimen, at least within the accuracy of the readings in this set-up ($\pm$.01 mm), and did not change throughout the test period. With the 10 kg loads, deformation increased up to 10′ after application of the load and thereafter appeared to remain constant. These findings led to the preliminary conclusion that total deformation under a constant stress is approached asymptotically, and deformation is a function of stress duration. However, the experimental procedure did not permit determination of whether or not the specimens returned to original length after removal of the load. In addition, some features of the deformation under a constant stress could have been lost in the accuracy of the readings. These problems were surmounted in Part 2.

Part 2

Five tension specimens from five subjects were tested using the equipment described on page 15. Readings of deformation were accurate to 0.0005 mm. Two specimens were loaded successively with 5 kg, 10 kg, then 20 kg and 30 kg loads. Three specimens were loaded successively with 10, 20, and 30 kg loads. All specimens were 3×2 mm in cross-section in the 25 mm reduced area. As with the earlier tests, the applied load was maintained for 30′, and then released. Deformation was recorded constantly

TABLE 11. *Changes in Deformation Over Time Under Differing Constant Loads*[1]

Subject	Load Kg	Loaded				Unloaded			
		Initial Deformation	5′	15′	30′	Initial Recovery	5′	15′	30′
1	5	24.0	24.5	24.5	24.5	0	0	0	0
	10	26.4	31.8	32.3	32.7	3.8	1.8	1.1	.7
	20	82.1	89.4	91.3	92.7	13.6	7.0	5.4	4.7
	30	154.2	182.6	192.5	198.9	42.4	26.9	24.1	23.2
2	10	55.0	59.8	60.5	60.6	5.1	3.5	2.6	0
	20	71.0	74.6	76.0	78.0	2.6	0	0	0
	30	98.7	104.1	106.3	108.2	11.8	3.9	2.5	2.2
3	10	83.3	86.3	86.9	87.4	3.4	0	0	0
	20	146.3	170.2	173.1	175.3	20.1	9.9	8.1	6.9
	30	207.4	231.9	247.4	261.4	51.8	29.6	26.1	24.7
7	5	11.5	11.9	12.0	12.0	.6	0	0	0
	10	14.6	14.7	14.8	14.8	1.2	0	0	0
	20	54.0	62.9	63.8	64.3	14.9	12.8	12.3	12.3
	30	92.7	99.2	101.0	102.0	18.7	14.5	14.5	13.4
19	10	28.8	30.3	30.9	—	2.0	0	0	0
	20	94.8	103.3	105.7	107.5	17.7	4.8	0	0
	30	146.0	187.4	210.3	232.3	90.0	64.0	60.1	58.3

[1] Deformation units are in micra, and are the difference between a recorded value and original length.

both graphically (Figure 23) and numerically via the voltmeter printing readout. If a specimen returned to original length, the test was deemed complete. If the specimen did not return to original length, recordings were continued for 30′ after unloading. Ten minutes were permitted between tests.

The results summarized in Table 11, and illustrated in Figure 23, were as follows: Under the constant load of 5 kg, 2 specimens attained maximal deformation almost immediately after application of the load with no increase noted for the remainder of the test period. Upon release of the load, they returned to original length, with most of the recovery being attained at the first instant of unloading, and the remainder within a few minutes.

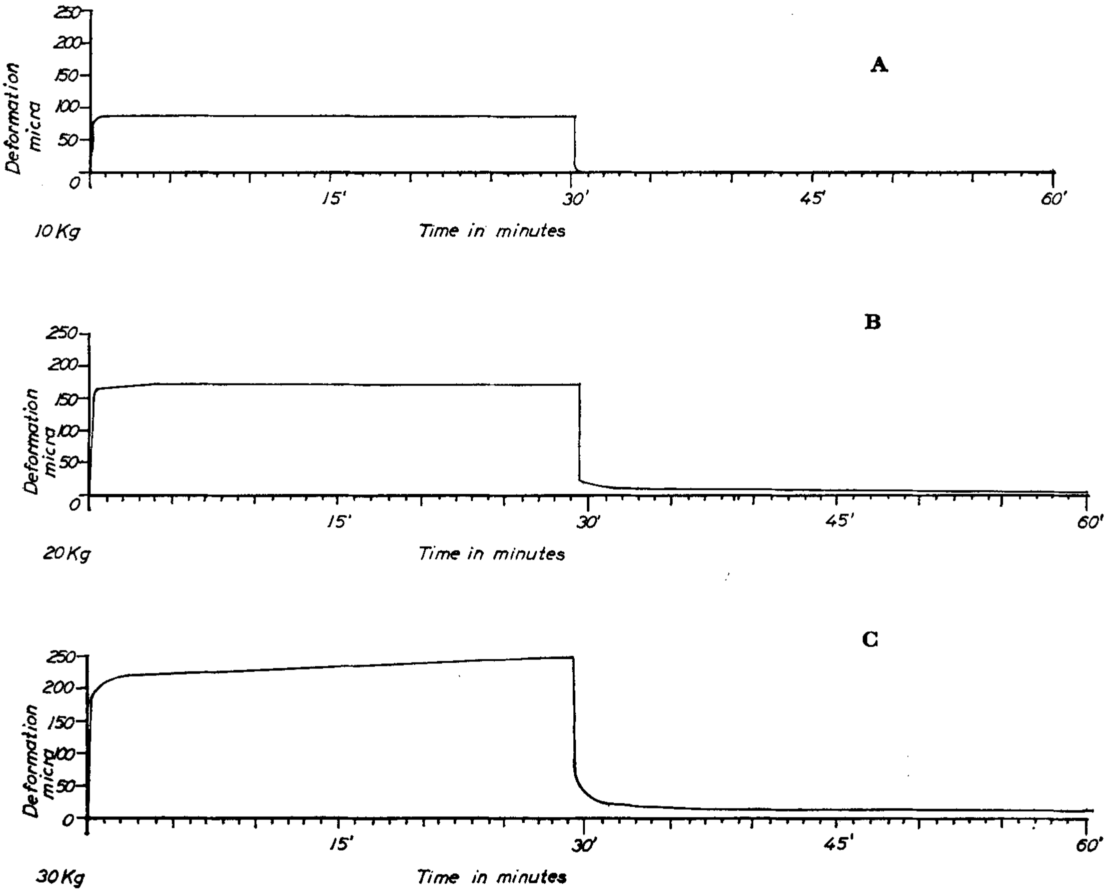

Figure 23. Changes in deformation over time under differing constant loads in one specimen. **A.** The behavior with a 10 Kg load. It is seen that the final deformation is attained quickly, and when the load is released at 30′, complete recovery takes place. **B.** A 20 Kg load. There is a definite delay in the attainment of a constant deformation. When the load was released at 30′, the specimen did not regain its initial length. **C.** A 30 Kg load has been applied. The specimen continued to deform for the entire 30′ load period. When the load was released after 30′, permanent deformation in seen. This deformation was superimposed upon that from the **B** test since the machine was rezeroed after test **B.**

With a load of 10 kg, the qualitative behavior began to vary. A definite retardation in attainment of total deformation occurred in some of the specimens. At this load, four specimens regained original length, and one did not.

With the load of 20 kg, additional changes were seen. All specimens showed progressive deformation up to 30′. With removal of the load, only one specimen regained original length within 30′.

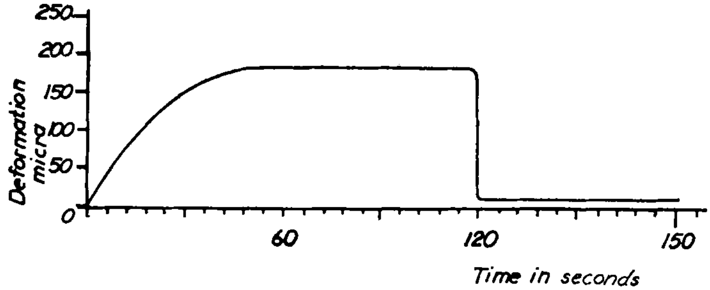

Figure 24. A rapidly recorded strain-time test. A 30 Kg load was applied and maintained for 2′ and released. It can be seen that there was delayed elasticity when the specimen was loaded at O, and instantaneous elasticity plus delayed elasticity when the specimen was unloaded. The difference in slope is due to a difference in the speed of loading and unloading as explained in the text.

The changes with the 30 kg load were similar to those of 20 kg, but more pronounced in that deformation was greater, and none of the specimens returned to original length within 30′.

The following features were common to all tests. The initial deformation was attained as rapidly as the specimens could be loaded (10—15″) with the equipment used, while the initial recovery length was attained as rapidly as they could be unloaded (1—2″). From this, it would appear reasonable to conclude that bone possesses the feature of instantaneous elasticity in addition to retarded elasticity. However, the feature of instantaneous elasticity cannot be immediately deduced from the curves in Figure 23, for the curve shape in constant recording apparatus is a function of recorder speed. The presence of this phenomenon was confirmed in additional tests on different samples from these subjects utilizing rapid recorder speeds and short term testing (Figure 24).

It was apparent that bone under certain constant loads will gradually yield, and when a greater load is applied greater yield is to be expected. This, feature varies from subject to subject. Analysis of the strain-time curves for all subjects showed the following equations to fit the data best.

$$\epsilon = A(1-e^{-Bt}) \text{ for loading, where}$$

A = a constant determining the ultimate deformation that will be attained, and
B = a constant determining the rate at which this deformation is reached.

$$\epsilon(t) = \epsilon_s + \epsilon_0' \, e^{-Bt} \quad \text{for unloading, where}$$

ϵ_s = some final deformation
ϵ_0' = deformation that remains after the instantaneous component of recovery
B = a constant for the rate of decrease in deformation

TABLE 12. *Deformation and Recovery from a Twenty-four Hour Constant Load*[1]

Subject	Initial Deformation	Loaded						Initial Recovery	Unloaded					
		Time in Hours							Time in Hours					
		1	5	10	15	20	24		1	5	10	15	20	24
1	109.0	137.7	143.9	144.4	146.3	147.1	147.1	[2]						
2	103.9	135.1	157.5	171.6	178.5	184.8	190.6	61.1	29.3	20.8	16.6	15.2	13.9	11.9
3	113.8	178.1	187.6	194.7	203.0	205.3	199.3	83.0	59.4	54.3	52.3	51.6	50.0	49.0
7	103.6	128.9	140.7	142.1	143.7	144.2	144.2	37.8	18.2	15.0	12.9	12.5	11.3	11.2
19	198.8	341.9		Fracture										

[1] Deformation is recorded in microns. In the unloaded half of the table, the deformations are to be interpreted as the remaining deformation at the time cited.

[2] This test had to be discontinued at this time to permit electrical work in other parts of the laboratory.

Yet, it could not definitely be stated on the basis of these data that the final deformation under the higher loads would be approached asymptotically as the loading equation implies or that the deformation resulting from the higher loads would not eventually be recovered. Longer term tests were required.

Part 3

Five additional specimens from the same subjects as in part 2 were used. The test set-up and specimen size were the same. A 30 kg load was applied, and maintained for 24 hours, then released. The recovery period was recorded for an additional 24 hours.

Results of these longer-term constant load tests at selected times are presented in Table 12. Two specimens attained a constant deformation within 24 hours, two continued to deform for the entire period, and one specimen failed after one hour. In the three specimens in which a recording of unloading was obtained, all showed persistent deformation at the end of the 24-hour recovery period. The data from the 2 specimens that showed continuing deformation for the 24-hour load period indicated that a constant deformation would be reached in the future. In the recovery recordings, it was seen that some further decrease in deformation would have taken place, but that original length would not be regained.

It can be concluded from this phase of the experiment that bone will attain a constant deformation under a constant load or will fail. If the load is of sufficient magnitude, permanent deformation will result.

Analysis

With the experimental evidence presented, the steps by which the mechanical model for bone was developed can be undertaken. This was done in stages. First, load-deformation curves of tests reported in Chapter III were analyzed with a view toward a mechanical model. The second step consisted of review of the specific experiments. Next, a model was postulated that was compatible with all of the information at hand. The chief goal was to obtain the simplest model that could account for all of the behavior observed.

Inspection of all load-deformation curves in the test specimens revealed one common feature for all axes of loading, rates of loading, stages of hydration, etc. *There were no straight lines.* (Figure 25.) Granted, that in certain instances the deviation from linearity was not great, the basic fact is unchanged. This feature eliminated the perfectly elastic body. In addition, the fact that there was a progressive increase in stress until failure of the specimens eliminated the perfect plastic and rigid plastic bodies from consideration. A simple Newtonian liquid did not have to be considered. The elimination of these fundamental bodies was implicit from the start, but it was necessary to consider them since, for example, the Hookean body could have represented a close approximation of a working model.

That elastic behavior is a feature of bone was evident from the cycle and creep studies — most of the attained deformation was recovered.

The presence of viscous damping of some type in bone could be deduced from several features — 1) the fact that deformation up to some constant load is some function of the deformation rate. 2) the fact that total deformation under a constant stress was not attained instantaneously, i.e. bone has a retardation time. 3) the fact that bone has a relaxation time. 4) the fact

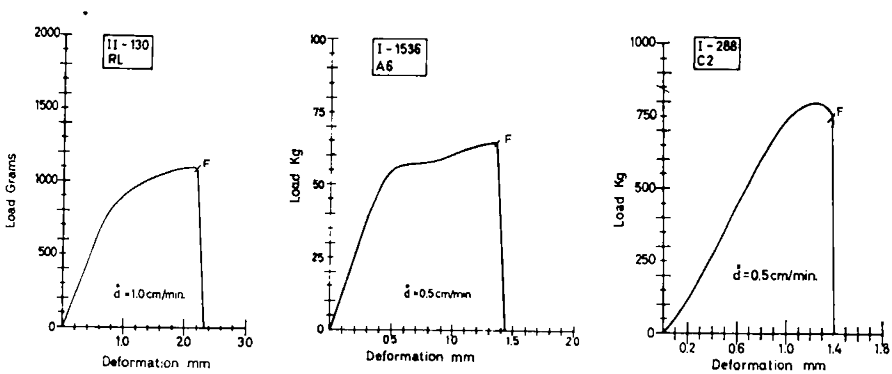

Figure 25. Typical load-deformation diagrams to failure. Left — Bending, end supports, center load. Center — Tension. Right — Compression of a $5 \times 10 \times 15$ mm block. The rate of deformation ($\dot{d}$) is stated with each curve. Complete failure is indicated by F.

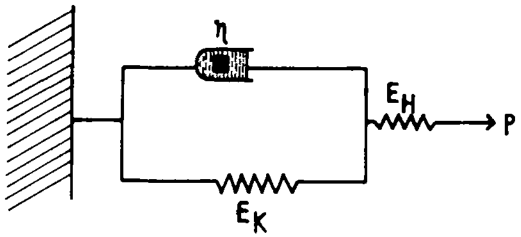

Figure 26. A possible rheologic model for cortical bone. It is shown as a Hooke element combined with a Kelvin element. This behavior could also be represented by an Hooke element in parallel with a Maxwell element.

that an hysteresis loop was present on load-unload cycle testing. Also, the shape of the usual load-deformation curve, although not conclusive in and of itself is strongly suggestive.

The Kelvin and Maxwell bodies were then reconsidered since they are the simplest bodies containing both an elastic and viscous element. The Kelvin body has a retardation time in common with bone. However, it does not have a relaxation time. From eq. 4, put $\epsilon=\text{constant}=\epsilon_0$, then $d\epsilon/dt=0$ and

$$\sigma=E\epsilon_0$$

The Maxwell body possesses the feature of instantaneous elasticity and a relaxation time in common with bone. However, under a constant stress, deformation continues to increase, whereas the constant load experiments showed that deformation became constant. In addition, it was shown in eq. 9, that with a constant deformation, stress relaxes to zero, rather than to a new lower level as demonstrated in the stress-relaxation studies.

At this stage, it was necessary to postulate a working model to account for some of the discrepancies noted, and to attempt to fit this model to the data. A model that appeared to account for the behavior at smaller loads is shown in Figure 26. It consists of a Hooke body linked in series to a Kelvin body. Introduce the subscripts $_H$ and $_K$ to denote Hookean or Kelvin behavior. The relation between stress and strain in this model is governed by the following equation:

$$\frac{d\sigma}{dt}+a\sigma=b\frac{d\epsilon}{dt}+c\epsilon \tag{10}$$

where

$$a=\frac{1}{\eta}(E_H+E_K);\ b=E_H;\ c=\frac{1}{\eta}E_HE_K$$

Eq. (10) is the constitutional equation of the model.

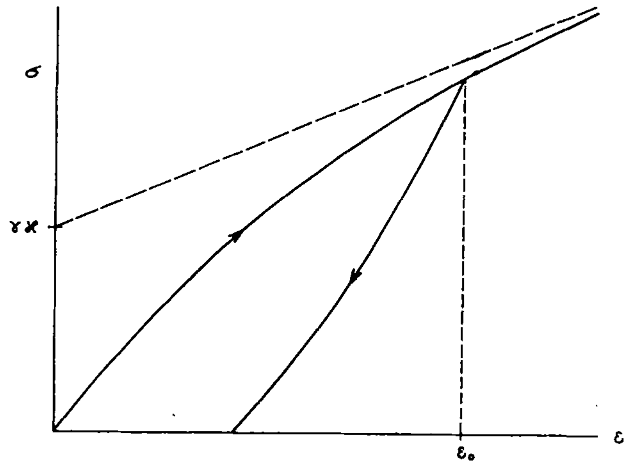

Figure 27. Diagram to demonstrate the behavior of the model, Figure 26, at a constant rate of deformation and unloading from, a certain deformation, ϵ_0.

First, it was necessary to analyze this model in relation to the experiments cited. Consider the case of loading and unloading with a constant rate of deformation. The time derivative $d\epsilon/dt$ is now constant, say $\varkappa$, and stress becomes a function of strain. Eq. 10 yields the solution

$$\sigma=\gamma\varkappa\left(1-e^{-\frac{a}{\varkappa}\epsilon}\right)+\frac{c}{a}\epsilon \tag{11}$$

where

$$\gamma=\frac{ab-c}{a^2}$$

Eq. (11) thus shows the relation between σ and ϵ for this case, and from Fig. 27, it is seen that the $\sigma-\epsilon$ curve approaches a straight line asymptotically, the slope of which is c/a or $E_K E_H/E_K+E_H$. This was in agreement with the experiments.

Now consider the case of unloading from a certain deformation in Figure 27 at a rate of deformation $d\epsilon/dt=-\varkappa$. From equation (10) the following solution is obtained:

$$\sigma=\gamma\varkappa\left[\left(2-e^{-\frac{a}{\varkappa}\varepsilon_0}\right)e^{-\frac{a}{\varkappa}(\varepsilon_0-\varepsilon)}-1\right]+\frac{c}{a}\epsilon \tag{12}$$

Eq. (12) satisfies the down curve of Figure 27, and the model therefore satisfied this behavioral feature of bone.

Next, the model was considered in relation to the data showing that the deformation of specimens decreased with a more rapid rate of loading and that the hysteresis loops would tend to close with rapid and very slow rates of deformation. From Eq. (11) it can be seen, that as the rate of deformation approaches zero, i.e. $\varkappa \cong 0$, the relation

$$\sigma = \frac{c}{a}\epsilon \tag{13}$$

is obtained which is a straight line, the slope of which is dependent upon the modulus of elasticity of the springs. On the other hand, if the rate of deformation is very rapid, i.e. $\varkappa$ is very high in Eq. (11), $e^{-\frac{a}{\varkappa}\varepsilon}$, could be expanded in powers of $\left(-\frac{a}{\varkappa}\epsilon\right)$ and neglecting higher powers, then the expression

$$e^{-\frac{a}{\varkappa}\varepsilon} \cong 1 - \frac{a}{\varkappa}\epsilon$$

is obtained which yields

$$\sigma = \left(\gamma a + \frac{c}{a}\right)\epsilon = b\epsilon = E_H\epsilon \tag{14}$$

which is a straight line with slope E_H.

It is thus seen that with a rapid rate of deformation, the Kelvin body in the model will not deform and the total deformation in this instance will be less for a specific load than for the same load applied slowly, where both elements can deform. Recall again that if units are connected in series they take the same load and deformation is additive. It follows then, that a modulus of elasticity calculated for the material will be higher in a rapidly performed test than a slowly performed test up to the same load. This agreed with the experiments on page 54.

A similar set of solutions can be derived if one postulates a constant rate of loading rather than a constant rate of deformation. From eq. (10), put $d\sigma/dt = \text{constant} = \beta$ and the solution is

$$\epsilon = \delta\beta\left(e^{-\frac{c}{\delta\beta}\sigma} - 1\right) + \frac{a}{c}\sigma \tag{15}$$

where

$$\delta = \frac{\eta}{E_K{}^2}$$

This would yield a $\sigma - \epsilon$ diagram similar to Figure 27.

Another feature that must be accounted for by the model is that of relaxation of stress under a constant deformation as shown in the stress relaxa-

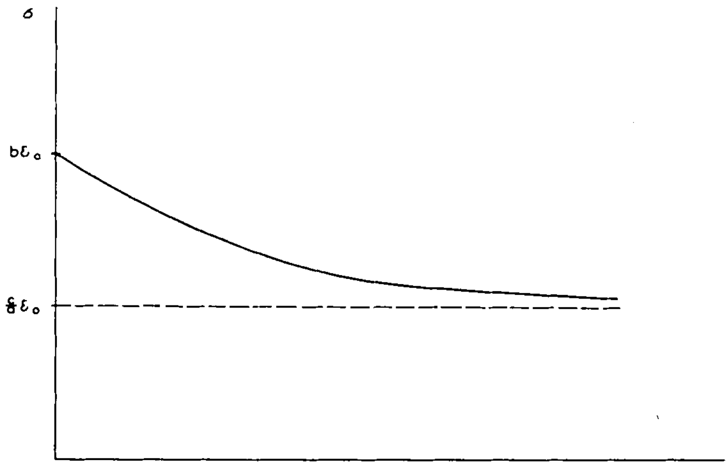

Figure 28. Diagram to demonstrate the predicted behavior of the model, Figure 26, in the case of a constant deformation, ϵ_0. Stress is plotted against time. The original stress, $b\epsilon_0$, decreases to the level $\frac{c}{a}\epsilon_0$.

tion experiments. From Eq. (10) put ϵ=constant=ϵ_0 and $\frac{d\epsilon}{dt}=0$. Eq. (10) then yields

$$\sigma=\epsilon_0\left(\gamma a e^{-at}+\frac{c}{a}\right) \tag{16}$$

which satisfies curves of Figure 21. It is seen that the stress will be asymptotic to a level determined by the constant strain and the moduli of elasticity of the springs i.e. $c\,\epsilon_0/a=E_K E_H\,\epsilon_0/E_K+E_H$, and will not tend to zero. This was in agreement with the stress-relaxation experiments and is shown in Figure 28.

The final feature to be considered was that of deformation over time under a constant stress. With the small loads it was shown that 95% of the total deformation under the constant stress conditions was attained instantaneously for practical purposes, and further increases in deformation appeared to be asymptotic to some level after about ten minutes. It is perhaps simpler to review the behavior of the components under this condition. For the Hooke body with a constant stress=σ_0, there is no time dependent relationship, so that an instantaneously applied stress will result in an instantaneous strain, and a strain-time diagram will be a straight line parallel to the time axis. For the Kelvin body, if at $t=0$, when the deformation $\epsilon=0$, a constant stress σ_0 is applied,

$$\epsilon_K=\left(\frac{\sigma_0}{E_K}\right)\left\{1-e^{\frac{E_K t}{\eta}}\right\} \tag{17}$$

so the strain (σ_0/E_K) which would occur instantaneously without the dash-pot, is approached exponentially. The following is thus obtained:

$$\epsilon = \frac{\sigma_0}{E_H} + \left(\frac{\sigma_0}{E_K}\right)\left\{1 - e^{-\frac{E_K t}{\eta}}\right\} \tag{18}$$

so that the strain (σ_0/E_H) is attained instantaneously and final strain asymptotically approaches the level $\sigma_0/E_K + \sigma_0/E_H$. This predicted response for the model agreed with that derived from the data on page 59. If as in the creep studies, the σ_0 is continued until deformation ϵ_0 is attained, and then released quickly, the strain at time t, will be composed of two components as above; that in the Hooke body which is recovered instantaneously, and that which is recovered exponentially according to

$$\epsilon_K = \left(\epsilon_0 - \frac{\sigma_0}{E_H}\right) e^{-\frac{E_K t}{\eta}} \tag{19}$$

Therefore, the deformation will ultimately be recovered.

However, it was seen in the creep studies that at 30 kg loads, none of the specimens regained original length after release of the load. The model postulated for lower loads, did not permit explanation of this feature and therefore an additional mechanism had to be added to account for this. Since deformations tended to become constant with all of the loads in the creep studies, this was evidence against the inclusion of an additional viscous damper in series. Additional evidence against a viscous damper in series was cited earlier — at low loads, all deformation was recovered.

Therefore it was logical, that the residual deformation was due to plastic flow and that a plastic friction element had to be added in series. It was necessary to include multiple friction elements in series since many of the stress-strain diagrams in the study showed a definite increasing load requirement to produce failure after a region of plastic flow (Figure 25 B). This type of behavior can be represented in a mechanical model by a series of frictional contacts loosely connected by strings (*Jaeger*, 1956). If these are appended to the model, Figure 29 is the result. The series of friction

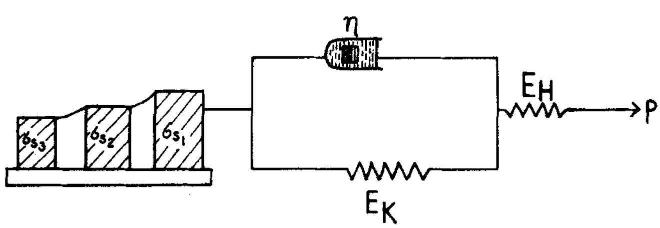

Figure 29. The working model, Figure 26, has been modified to include three frictional elements, σ_{s1}, σ_{s2}, σ_{s3}, loosely connected by strings to account for plastic flow and work hardening.

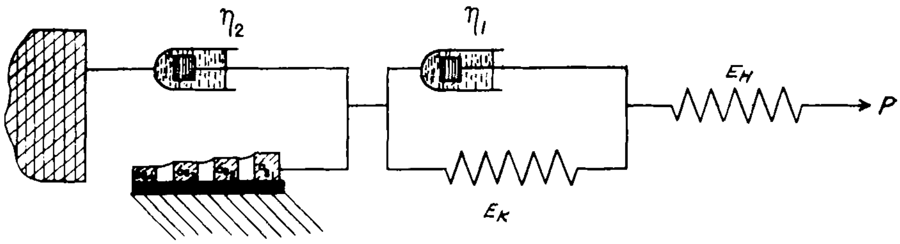

Figure 30. A further modification of the working model that accounts for damping of plastic flow.

contacts, each of which has a progressively smaller yield stress can be understood as acting in the following fashion. The force which is acting on the bone produces no permanent deformation until the yield stress of the first frictional contact is reached. If the force is then maintained at this level, the flow of the first frictional contact is limited by the yield stress of the second friction element, which will not deform until yield stress of the combined friction elements is reached. This continues in a stepwise fashion as the force increases. It is to be understood that the number of friction elements to be included has not been determined; thus Figure 29 is illustrative.

The appendage of the series of friction elements accounted for all of the data with one exception. In the creep studies, it was noted that there were no sudden increases in deformation after the initial load was applied i.e. the deformation increased gradually until a constant level was reached. If the friction elements were acting in an undamped fashion, then one would expect to see sudden increases at various phases of the strain-time diagrams as different yield stresses were overcome, with the resumption of increasing defôrmation as the Hooke-Kelvin element continued to deform. It was therefore necessary to damp the action of the friction elements. This can be done by adding another viscous damper in parallel. (Figure 30). The addition of this element thus prevents the friction elements from deforming instantaneously. Conversely, the deformation of the damper is limited by the yield stresses of the friction elements. There was however, no evidence to support the addition of a second viscous damper. In view of this, the model was rearranged so that the first viscous damper affected the spring and friction elements. (Figure 31). This model will act in the following fashion: as long as the stress in the spring E_K is less than stress in the first friction element, σ_{s_1}, then eq. (10) is valid. Above σ_{s_1}, the model behaves as a Maxwell body as the level $\sigma_{s_1}+\sigma_{s_2}$ is approached. If the stress does not attain $\sigma_{s_1}+\sigma_{s_2}$ deformation will become constant. This is repeated as successive yield stresses are encountered.

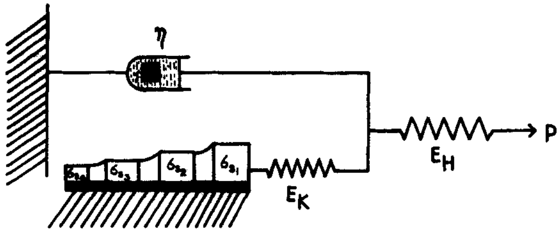

Figure 31. A Rheological Model for Cortical Bone. This is a *generic* model. A description is in the text. The constants for the springs are maintained E_H, E_K as in previous models, since there is Kelvin behavior below the yield stress of the friction element, σ_{s_1}.

This model (Figure 31) thus explains all of the observations, and answers the requirement of being the simplest model that was compatible with the available data. It consists of a Hooke body linked in series to a unit consisting of a Newton body in parallel with a modified Prandtl body. No attempt should be made to ascribe specific anatomical elements to the components of the model.

Recapitulation

The model developed here describes general mechanisms by which bone samples deform *in vitro*. It accounts for the properties of instantaneous elasticity, retardation time, relaxation time, plastic flow, and damped plastic flow, all having been observed in samples tested in this study. It can serve as a basis for the quantitative prediction of cortical bone properties.

The model provides a partial explanation for differences in physical properties that are noted when different conditions of testing are used, such as the effects of drying, temperature, and changing the rate of deformation.

A stress-strain diagram of dry-tested bone is more nearly linear than that of wet-tested bone. Stress-relaxation and hysteresis are decreased when bone is dry. In terms of the model, this is equivalent to stating that bone is more elastic when dry than wet. The effects of the viscous damping are vitiated by the drying, leaving in the model, a spring in series with friction elements or a perfectly plastic body. Under a progressive load at a constant rate of deformation, the expected behavior is that of dry bone tested to failure.

It was shown that increases in deformation were present when samples were tested at 37° C as contrasted to 21° C. This temperature-dependent behavior is expected in substances possessing viscous properties and is therefore expected behavior in bone in view of the presence of a viscous damper in the model.

The amount of deformation in a sample varied with the rate of deformation, the samples deforming less at a more rapid rate of deformation than with a slower rate. In addition, deformation varied with the duration of stress, and stress varied with the rate of deformation. It was predicted that stress should vary with the rate of stress. There are many possible combinations of these variables, all of which would result in differences between calculated properties.

In the derivation of the model, previously undisclosed facets of the physical properties of bone were delineated. Other features were confirmed. Essentially, this work has demonstrated that it is possible to describe the behavior of bone in terms of a mathematical model. With this as a matrix, a different approach to the study of the physical properties of bone can be undertaken. Heretofore, the ultimate strength and modulus of elasticity have been the most frequently studied properties of bone. The properties of stress relaxation, strain retardation, instantaneous elasticity, and yield stresses have received relatively little attention. It was shown that the modulus of elasticity is not constant. Similarly, it is possible that the ultimate strength of bone may change with the rate of deformation, (*McElhaney*, 1965), rate of stress, and other potential variables. The same would apply to the other properties just mentioned. Therefore, it becomes apparent that rigid standards of testing are needed for the comparison of data from different sources, and for the incorporation of the study of the physical properties of bone into the realm of medical diagnosis.

In this study, the constants for elasticity, viscous damping, and yield stresses were not derived, since it was felt that they should be based upon mean values obtained from a large group of "normal" subjects. With the determination of the model constants, it will be possible to predict the reaction of bone to mechanical forces under conditions otherwise difficult or impossible to simulate in a laboratory. For the purpose of prediction, it would be better to devise an electrical analogue for the mechanical model shown here. The appropriate electrical equivalents exist, (*Stacy*, 1955; *Papazian*, 1962) and their use facilitates the confirmation or prediction of experimental results (*Stulen*, 1962), and would support further investigations of this type.

CHAPTER V

SUMMARY

A study of the physical properties of bone utilizing 663 samples of human femoral cortex obtained at autopsy from 43 subjects has been performed. The study was conducted under controlled laboratory conditions in which the effects of variables in the methods of preparation, methods of storage, methods of testing, ages of subjects, sources of samples and their microanatomy were examined. From this information, it was possible to derive for the first time, a model for the deformation of cortical bone using rheologic symbology and methodology.

CONCLUSIONS

1. In the studies related to the effects of variables in the selection preparation, storage, and testing methods, the following features, presented in more detail on page 43, were demonstrated.
   a. Freezing of samples has no effect on the physical properties of cortical bone. Alcohol and formaldehyde fixation may have some effects.
   b. Air drying changes the properties.
   c. Heating to 100–105° C produces marked changes in the properties.
   d. The permanent effects of heating when combined with drying are greater than with either variable alone.
   e. The deformation of wet samples varies directly with the temperature of testing.
   f. Wet samples can be loaded repetitively without permanent effects if small loads are used.
   g. There are significant differences in physical properties from subject to subject and within various locations in the femur. No differences could be attributed to age or Haversian canal volume.
2. The following properties were found to be present in cortical bone.
   a. Instantaneous elasticity.
   b. Strain retardation.
   c. Stress relaxation.
   d. Multiple yield stresses.

e. Implicit in a—d are the phenomena of: elastic fore-effect, elastic after effect, creep, hysteresis, and non-linear behavior on an usual load-deformation diagram, all of which were demonstrated in this study.

3. A rheologic model that accounts for the above features was derived. The model consists of a Hooke body, linked in series to a unit consisting of a Newton Body in parallel with a modified Prandtl body. The model confirms or predicts the following:

a. Deformation in bone samples is a function of the rate of deformation and the duration of the stress.

b. Stress in bone samples is a function of the rate of deformation and the rate of stress.

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From the Orthopaedic Hospital of the Invalid Foundation,
Helsinki, Finland (Head: Professor A. Langenskiöld, M.D.)

Coxa Plana

A CLINICAL AND RADIOLOGICAL INVESTIGATION WITH PARTICULAR REFERENCE TO THE IMPORTANCE OF THE METAPHYSEAL CHANGES FOR THE FINAL SHAPE OF THE PROXIMAL PART OF THE FEMUR

WALTER EDGREN

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