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MEASUREMENT OF STABILITY OF TIBIAL FRACTURES

A Mechanical Method

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I. Introduction

Whether and to what extent union of a fracture is taking place is reflected in the stability of the fracture. To determine whether this has increased over a given period is, however, by no means a simple matter, especially in the case of delayed union, but it would seem that an objective measure of the stability is provided by the degree to which the fracture is deflected under a load. To devise a method for following the development of stability by measuring this deflection at intervals was the object of the investigation reported in this monograph.

In principle, the deflection of the fractured bone is recorded as a function of the applied bending force, and an instrument has been designed by means of which such a registration can be made. The reliability of the instrument has been determined experimentally on autopsy specimens of the tibia, and the suitability of the method for clinical use has been examined by performing measurements on patients.

The deflection of a bone produced by the application of a load is dependent on its modulus of elasticity. This quantity, which affords a basis for comparison of the stiffness of bones, can be calculated from the deflection of the bone and the moment of inertia of its cross section. As no picture of the section can be obtained in the living subject a formula has been derived that provides an approximate value of the moment of inertia of this area. The required dimensions of the cross section are determined from radiographs taken with known projections.

Thus, from the deflection of the bone and the moment of inertia a rough measure of the stability of the fracture can be obtained. The total deflection of the bone is composed of the deflection of the bone fragments and that within the actual fracture. The former component is obtained from measurements on the other, sound, tibia, and by subtracting this from the total deflection, the deflection within the actual fracture is obtained, and hence the flexural rigidity of the fracture callus can be determined. This rigidity reflects the stage of healing and hence indicates when the various steps of treatment should be introduced—for example, when it is safe to permit normal functional loading of the leg.

II. Physical properties of bone

Since Wertheim's studies published in 1847 the physical properties of bone have been a subject of increasing interest, and many of them have been elucidated in reviews published in recent years (Evans 1957, 1966; Knese 1958; Blaimont 1968).

Bone being a heterogeneous anisotropic substance with varying structure, the determination of its deformation and moment of inertia presents considerable difficulty; but neither of these properties need be known for the ultimate strength to be found. In tension and compression the stress is distributed uniformly over the cross sectional area and if tensile and compressive tests can be used to obtain values for the required mechanical properties of the bone, a knowledge of the moment of inertia is unnecessary; but if bending or torsional tests are used, this moment must be known, because, like the modulus of elasticity or rigidity, it is a critical quantity for the magnitude of the deflection or torsion.

If test specimens can be used instead of large pieces or intact bones, the moment of inertia can be accurately calculated from the cross section. Most studies of the mechanical properties of bone have been performed on such test specimens (Olivo *et al.*, 1937; Carothers *et al.*, 1949; Dempster & Liddicoat, 1952; Evans, 1957; Dempster & Coleman, 1961; Vose, 1962; McElhaney, 1964; Sedlin, 1965; Sweeney *et al.*, 1965; Sedlin & Hirsch, 1966; Lindahl & Lindgren, 1967). Where large pieces of bone or intact bones have been used, it has been values for the properties in tension or compression or for the ultimate strength that have been determined; otherwise the moment of inertia of the section must have been calculated.

In determinations of the moment of inertia the cross section of the bone has been taken as circular or elliptical (Grunewald, 1920; Bell *et al.*, 1941; Weir *et al.*, 1949; Vose & Kubala, 1961; Currey, 1969). A numerical integration of the moment (see page 33) after balancing the centre of gravity has been performed and the principal central axes have been inserted by inspection (Koch, 1917). Blaimont (1968) has determined the moment of inertia in a like manner after finding the direction of the axes experimentally. By graphic integration Marique (1945) found the moment of inertia from calculated directions of the principal central axes. From photographic enlargements of the cross section of a bone Mather (1967) calculated its moment of inertia by a graphical method devised by Mohr (Andrews, 1934). Knese *et al.* (1956) determined the moment of inertia and the directions of the principal central axes of bone specimens by means of an integrator, as did Comtet *et al.* (1967).

The elasticity and strength of bone vary according to the type of deforming force and also according to the relative position of the longitudinal axis of the test specimen and axis of the bone. The elasticity and strength are least in the transverse direction and greatest parallel to the axis (Olivo *et al.*, 1937; Dempster & Liddicoat, 1952; Dempster & Coleman, 1961; Sedlin, 1965, Sedlin & Hirsch, 1966; Hirsch & da Silva, 1967; Blaimont, 1968; Lang, 1969). This is due to the largely axial orientation of the collagen fibres; these have a high tensile strength and modulus of elasticity for axial tension. Apatite, whose crystals are oriented with their longitudinal axes parallel to the collagen fibrils, has a low tensile but high compressive strength, and a higher modulus of elasticity for axial compression (Glimcher, 1959; Molnar, 1960; Currey, 1964; Sweeney *et al.*, 1965).

According to three current theories on the combined function of the collagen and the apatite these components can act as a compound bar, the collagen fibres can be pre-stressed so that the bone behaves as a pre-stressed material (Knese, 1958), or the bone can constitute a two-phase material similar to fibre glass (Currey, 1964; Mack, 1964).

The modulus of elasticity is slightly higher for tension than compression. When a bone is bent the stress and strain are compressive on the concave side and tensile on the convex, and the modulus of elasticity in bending will be a resultant of the values for compression and tension. There is a close relationship between the maximum bending strength and the modulus of elasticity in bending, which is the parameter that can be used in clinical work for comparing different vital bones (Weir *et al.*, 1949; Smith & Walmsley, 1959; Sedlin & Hirsch, 1966; Mather, 1967).

Values of the modulus of elasticity in bending that have been reported for the human tibia include $(0.81—1.42)\times10^3$ (Smith & Walmsley, 1959), 1.22×10^3 (Motoshima, 1960), $(1.4—2.6)\times10^3$ (Rabischong, 1965), and 1.44×10^3 kgf/mm^2 (Mather, 1967); and for the femur, $(1.06—1.97)\times10^3$ (Sedlin & Hirsch, 1966), and 1.36×10^3 kgf/mm^2 (Mather, 1967).

III. *Notation and formulae*

The following notation has been adopted in this monograph.

σ	Stress	M	Bending moment
ε	Strain	P	Concentrated load
v	Deflection recorded under load	a	Diameter of marrow space
δ	Corrected deflection	b	Breadth of section of the tibia
E	Young's modulus of elasticity	d	Thickness of section of the tibia
I	Moment of inertia	l	Distance from nearer outer pin to fracture
L	Length, span		

FORMULAE

The derivations of the formulae used in this study are to be found in the usual reference books of mechanics (Andrews, 1934; Baumeister & Marks, 1958). In the following theoretical analysis of the deflection of the tibia it is assumed that bone is an ideal elastic material.

The application of a force to an elastic material produces a stress σ that is proportional to the strain ε in accordance with Hooke's law:

$$\sigma = E\,\varepsilon \qquad [1]$$

where E, the modulus of elasticity, is a constant that is valid below the limit of proportionality. It can be calculated from the expression for the deflection of a uniform straight beam freely supported at both ends and subjected to a load at its midpoint. The formula, which is based on Mohr's theorem, is

$$\delta = PL^3/48\,EI \qquad [2]$$

All the values of the modulus of elasticity stated in this report refer to bending.

The positions of the principal central axes are obtained from the equation:

$$\tan 2\,\varphi = (I_{135^\circ} - I_{45^\circ})/(I_y - I_x) \qquad [3]$$

where I_x and I_y are the moments of inertia of the tibia section about the axes of a rectangular coordinate system xy with the origin in the centroid, and where φ is the required angle between one principal central axis of the tibia section and the axis of x. I_{45° and I_{135° are the moments of inertia about the axes of a coordinate system having the origin in the centroid but rotated through 45° in relation to the coordinate system xy.

The principal moments of inertia of the tibia section are given by the formula:

$$\begin{matrix} I_{max} \\ I_{min} \end{matrix} = \frac{1}{2}\left\{ (I_x+I_y) \pm \left[(I_x-I_y)^2+(I_{135^\circ}-I_{45^\circ})^2\right]^{1/2}\right\} \quad .. \quad [4]$$

If the central axis is rotated through an angle φ about the centroid in the plane of section the moment of inertia will change to:

$$I\varphi = I_{min}\cos^2\varphi + I_{max}\sin^2\varphi \quad .. \quad .. \quad .. \quad .. \quad .. \quad [5]$$

Its magnitude is thus dependent on the angle φ and the magnitude of I_{min} and I_{max}.

The maximum deflection of a simple beam freely supported at the ends and loaded at its midpoint is obtained when the force acts against the minimum moment of inertia, that is to say, in the bending plane with respect to the minimum moment of inertia. If the line of action of the bending force P (Fig. 1) applied to the tibia, regarded as a simple beam,

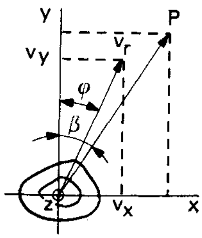


FIGURE 1 *Projections of the force P on the principal central axes x and y. The axis of the tibia coincides with the axis of z. v_x and v_y are the components of the deflection along the respective axes, and v_r is the resultant in the bending plane making an angle φ with the plane YZ, with which the line of action of P makes an angle β.*

does not lie in the bending plane YZ with respect to the minimum moment of inertia, the deflection will occur in the direction of a resultant v_r whose components v_x and v_y coincide with the axes x, y, the principal axes of the tibia section.

If the line of action of the force P makes an angle β with the plane YZ, one component of the force, $P \sin\beta$, acts along the axis of x, and the other, $P\cos\beta$, acts along the axis of y. The components of the deflection are thus $v_x = PL^3 \sin\beta/48\, EI_{max}$, and $v_y = PL^3\cos\beta/48\, EI_{min}$. The resultant v_r will act in a bending plane making an angle φ with the bending plane YZ, where:

$$\tan\varphi = I_{min}\tan\beta/I_{max} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad [6]$$

The magnitude of the total deflection v_r can be calculated from v_x and v_y by application of the Theorem of Pythagoras. The quantity recorded,

v_{obs}, is, however, the deflection in the direction of the force P—that is, $v_r \cos(\beta - \varphi)$. The maximum deflection is obtained if $\beta = 0$. The following relationship is obtained between v_{obs} and the required maximum deflection, v_{max}:

$$v_{obs} = v_{max} \left(\cos^2\beta + \sin^2\beta \; I^2_{min}/I^2_{max}\right)^{1/2} \cos(\beta - \varphi) \qquad [7]$$

MINIMUM MOMENT OF INERTIA OF THE TIBIA

Since the cross section of the tibia varies in shape and area throughout its length, the minimum moment of inertia of the section, $I_{min(x)}$, also varies. The midline of the tibia may be considered to be composed of the centroids of all the sections. The deflection of the midline under load is dependent on the moment of inertia, a quantity included in eqn. [2] for the deflection of a simple beam loaded at its midpoint, $\delta = PL^3/48\,EI$. The product EI—the flexural rigidity—varies from one bone to another, and of these two factors determining the deflection I is the one that displays by far the greater variation.

The outer contour of the cross section of the tibia is roughly triangular in shape, with rounded corners and slightly convex sides. The base of this triangle is regarded as lying in the medial surface of the tibia (Fig. 2). The

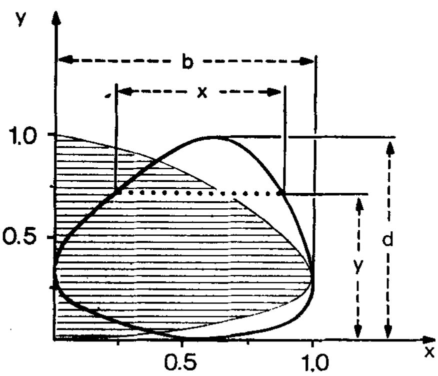


FIGURE 2 *The standardized section in which the width x/b and the depth y/d are plotted on the axes of x and y respectively. The moment of inertia of the shaded area so obtained is the same as that of the standardized section.*

moment of inertia of a triangle about the axis through the centroid parallel to the base is given by the expression $I = bd^3/36$, where b is the base and d the height of the triangle. If the cross section of the tibia is regarded as a triangle, its moment of inertia will be determined chiefly by the depth d in the plane of the section, since this is raised to the power of three.

The moment of inertia of the tibia can be calculated from the dimensions measured on radiographs taken in two projections. With the central ray in the plane of the medial surface the depth of the cross section (d) and the diameter of the marrow space (a) are measured. The projection perpendicular to this gives the width of the tibia (b), but not, however, the dimensions of the marrow space, because it is masked by the fibula.

To examine whether these measures can be used to calculate the moment of inertia to a high enough level of accuracy the following procedure was used.

Forty-five cross sections from the narrowest portion of the tibia were examined and from stamped impressions of each the outer contour was obtained, which was then standardized. Let the greatest width be b and the greatest depth d. To obtain a standardized shape of each cross section the variation of the width x with y was measured. The relative width is expressed as the ratio x/b, and the corresponding relative depth as y/d. The values so obtained were inserted in a coordinate system with x/b and y/d on the axes of x and y, respectively.

It was seen that the standardized cross sections had almost the same appearance and it was possible to find a common formula for the moment of inertia.

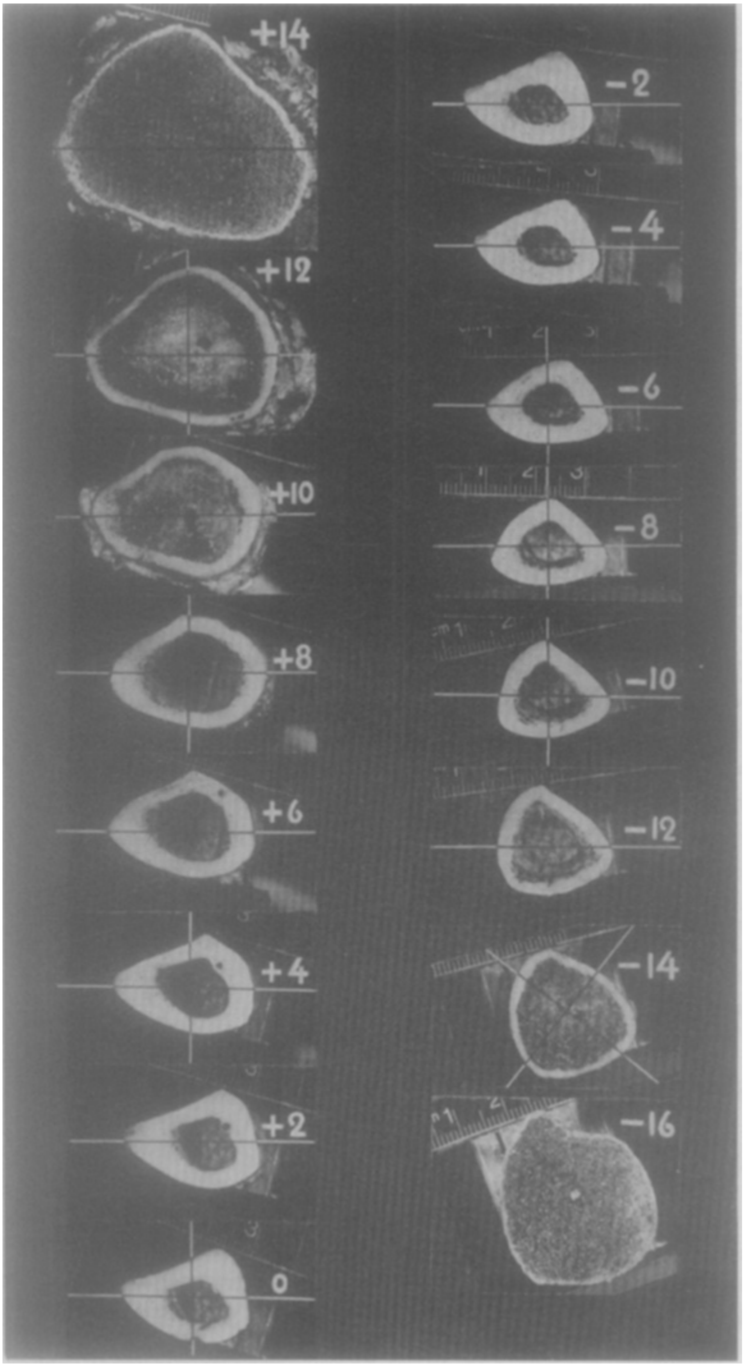
The moment of inertia was calculated for the central axis parallel to the side included in the medial surface and to the axis of x. The figure so obtained in which one boundary is the axis of y and the other the curve defined by the points having x/b and y/d as co-ordinates—the shaded area in figure 2—has a moment of inertia that is equal to that of the standardized section. This is calculated as the sum of all the elemental area multiplied by the square of their distance to the central axis—in this case 0.042. The expression for the moment of inertia of the area defined by the outer contour of the tibia is 0.042 bd^3. The cross section of the marrow space was taken a as circle of diameter a. The moment of inertia of a circle about the central axis is $\pi a^4/64 = 0.049a^4$. The approximate expression for the moment of inertia of the section is thus

$$I_{appr} = 0.042\, bd^3 - 0.049\, a^4 \quad \ldots \quad [8]$$

The displacement of the centroid on subtraction of the moment of inertia of the circle is so small that its effect on the remaining moment of inertia is negligible.

DEFLECTION OF THE TIBIA

The shape of the cross sections varies at different levels of a tibia (Fig. 3). Because the moment of inertia also varies throughout the length of the tibia, the constant 1/48 in eqn. [2] does not apply to this bone. The


+14
+12
+10
+8
+6
+4
+2
0
-2
-4
-6
-8
-10
-12
-14
-16

minimum moment of inertia of the cross sections of 5 tibias are shown at the top of figure 4. For one of the bones the moment was determined by means of integrator, and for the other by numerical integration. The minimum moment of inertia of the tibia I_{min} and that of each section $I_{min(x)}$ is placed on the axis of ordinates and is plotted as a function of the length of the tibia. Thus, the curve for each bone shows the variation of I_{min} along the tibia. From the values in each curve the deflection curve for each specimen was calculated by means of a computer. At the bottom of figure 4 one such curve—for specimen 2, left tibia—is compared with that which would be obtained if the tibia were a straight uniform beam. At the same time as the curves were computed the value of the constant in eqn. [2] was obtained for each of the five bones and the two loading systems with the supporting pins 21 and 24 cm apart (see page 19). As regards the minimum moment of inertia for the smallest cross section, the constant in the formula for the deflection of the tibia will be 1/66 and 1/67, respectively. The standard deviation of the constants for the five bones is 5 per cent. The expressions for the deflection of the tibia are thus:

$$\delta_{21}=21^3P/66\ EI_{min} \quad \delta_{24}=24^3P/67\ EI_{min} \quad \ldots \quad [9]$$

The original eqn. [2] was used in the subsequent calculations for determining E, and the values of the modulus of elasticity so obtained are designated the apparent modulus of elasticity, "E", while the values yielded by eqn. [9] are given below together with the magnitude of the constant.

DEFLECTION OF THE CALLUS

Fracture callus may be regarded as a visco-elastic material. During the course of the union the viscous component diminishes and the elasticity of the callus increases until it is ultimately the same as for the rest of the bone. This may be checked by measuring the distance between the rising and descending part of the load-deflection curve when the applied force is zero.

A true model for the deflection of the callus would thus require the application of advanced plastic theory, in which the viscous and elastic component are functions of the time. Account would also need to be taken

FIGURE 3 *Cross sections taken from 14 cm proximally (+) to 16 cm distally (—) of the midpoint (0) of a tibia specimen. The intersecting lines are the principal central axes of the moment of inertia, whose positions were determined by means of an integrator. A single line through the section marks the direction of the principal central axis of the minimum moment of inertia, which was inserted by inspection.*

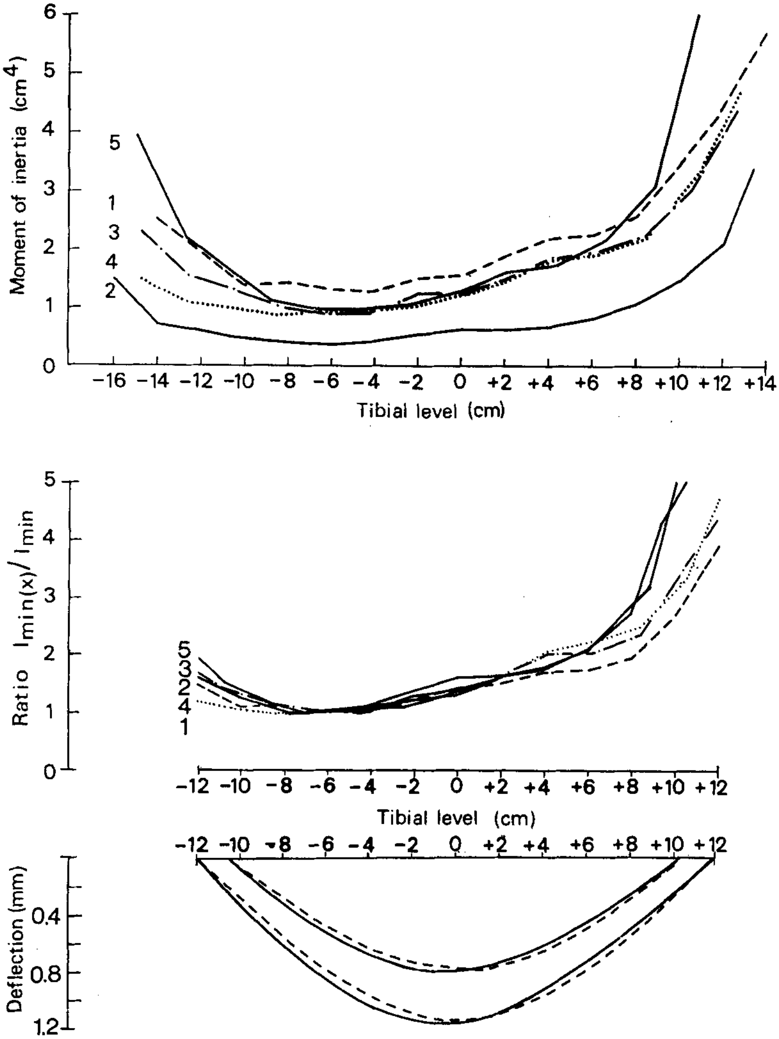


FIGURE 4 *Top.—Variation of the minimum moment of inertia of the sections along the tibia. Specimens 1—5.*

Middle.—Variation of the ratio $I_{min(x)}/I_{min}$ along the tibia for the same specimens.

Bottom.—Deflection of specimen 2 (continuous line) and of an ideal beam loaded by a force of 5 kgf, with the end supports 24 and 21 cm apart (broken line).

of the shear accompanying bending. As such a model could not be applied in practice and would be understood only by specialists in the mechanics of materials, a simple model was chosen where the callus is first regarded as a spring joint, a joint which can take up a bending moment, and later as a very short beam, where the viscous component and shear are neglected. This approach is necessary in order to obtain a conception of the physical properties of the callus.

In the model, the fractured bone is regarded as two beams connected by a spring joint; the applied bending moment of the spring joint is given by the expression $M = C\alpha$, where C is the spring constant and α the angulation of the spring joint. The total deflection of the model δ_{tot} is the sum of the deflections of the beams δ_t and the spring joint δ_f, that is,

$$\delta_{tot} = \delta_t + \delta_f$$

The angle α can be obtained in terms of δ_f and l as follows (Fig. 5).

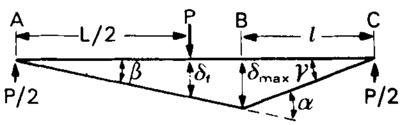


FIGURE 5 *A fractured tibia represented as a beam AC on two end supports distance L apart, with a concentrated load P. B indicates the fracture at distance l from nearer end support.* δ_f *is the displacement at the mid-point of AC;* α *is the angle of deflection at the callus, and* δ_{max} *the maximum displacement; P/2 is the reaction at the end supports.*

From the theorem relating to the external angle, $\alpha = \beta + \gamma$. Since for small angles the tangent is nearly equal to the angle itself, $\beta = \delta_f/(L/2)$ and $\gamma = \delta_{max}/l$. But, from the theorem on similar triangles, $\delta_f/(L/2) = \delta_{max}/(L-l)$; thus $\delta_{max} = 2\delta_f\,(L-l)/L$. Hence, $\gamma = 2\delta_f\,(L-l)/Ll$. Inserting these values of β and γ in the equation $\alpha = \beta + \gamma$, we have

$$\begin{aligned}\alpha &= 2\delta_f/L + 2\delta_f\,(L-l)/Ll \\ &= 2\delta_f/l\end{aligned}$$

But the bending moment acting on the spring joint $M \rightarrow Pl/2$. Therefore,

$$C = Pl^2/4\delta_f \quad \ldots \quad [10]$$

where C is the stiffness of the callus in the model for the fractured bone. In the course of healing C increases inversely as the deflection of the callus δ_f. The equation is valid when the load-deflection curves of the fractured bones are linear, as they were in most cases at the stage of clinical stability.

It is of interest to see how the stiffness develops in the callus independent of the dimensions of the callus. In the model the spring joint is substituted by a beam of very short length s.

$C = M/\alpha$; but for a beam $\alpha = Ms/E_f I_f$, and hence $C = E_f I_f/s$. But since $C = Pl^2/4\,\delta_f$ and thus we have:

$$s/E_f = 4\,\delta_f I_f/Pl^2 \quad \ldots \quad [11]$$

If the three beams are taken to represent the fractured tibia, E_f, in eqn. [11], is the modulus of elasticity, I_f the moment of inertia and δ_f the deflection of the callus, which has a very short length of s.

δ_f is calculated in the following way. The bone fragments corresponding to the two beams have an observed deflection that is regarded as being equal to the observed deflection of the sound tibia, v_t. By subtracting v_t from the observed deflection of the fractured tibia, v_{tot}, the deflection of the callus is obtained. Thus,

$$\delta_f = v_{tot} - v_t \quad \ldots \quad [12]$$

The ratio s/E_f is a measure of the stability of the callus. Neither s nor I_f can be determined. Since the former moment of inertia of the bone should be recovered on healing, the moment of inertia of the callus, I_f, was considered to be equal to that of the sound tibia at the level corresponding to that of the fracture. It is obtained as a multiple of I_{min} from the curves for the ratio $I_{min(x)}/I_{min}$ (Fig. 4, *middle*).

When a fracture heals the deflection of the callus decreases and δ_f approaches zero; thus E_f approaches very high values.

STATISTICAL METHODS

In the evaluation of the results the usual statistical methods were applied and the procedures are to be found in *Statistical Methods* by G. W. Snedecor and W. G. Cochran (1967).

The results of the measurements on the right and left tibias or of different methods of measurements were compared by analysis of variance, and the probabilities, P, of hypotheses set up were determined by means of the F ratio. Student's t was used to determine confidence intervals for the observed means.

Linear regression analyses were performed by the method of least squares.

IV. Methods

MEASURING INSTRUMENT

The instrument consists of an aluminium beam (*4* in Fig. 6), referred to as the "measuring beam", three stainless steel pins (*1*, *2* and *3* in Fig. 6), a template *5* and a measuring cylinder *6* containing a force and a position transducer, which are connected *via* an electronic system. In each end of the beam are four holes, located 7.5, 9, 10.5 and 12 cm from its midpoint. A pin inserted in one of the holes can be locked in position. The central part of the beam is in the form of a cylinder; into this fits the measuring cylinder which can be moved in relation to the beam and fixed in selected position with the lever *9*. The cylinder holder also takes the template *5*, by means of which the central pin is inserted in its correct position.

Pins *1* and *3*, the outer pins, have a shaft 4 mm in diameter which ends in a needle 18 mm long and 1.5 mm in diameter. This is ground to a point on four sides to give a semi-angle of 10°; the extreme end of the needle has the form of a cone, to prevent it turning over on insertion into the bone (Fig. 7). The middle pin is essentially the same as the outer pins, except that its shaft is furnished with a guide and a thread so that it can be locked to the shaft of the template or the shaft of the measuring cylinder.

The cylinder constitutes a housing for a position transducer *10* and a force transducer *12*. The shaft *11* runs through the position transducer and is connected to the force transducer, to which is also connected a screw with a wheel *13*.

The position transducer (Bofors Type RLL-1) is designed for static and dynamic determinations of positions within ±12 mm, with a deviation in linearity of ±0.5 per cent. The sensory component consists of a differential transformer with a movable core connected in the shaft *11*. The calibration values given by the manufacturer are accurate to 0.5 per cent for the temperature range 10—30°C.

The force transducer (Bofors, Type KRK-2) has a measuring area of range 0 to ±10 kgf. The sensory component consists of a metal cylinder, whose axis is perpendicular to the direction of the applied force, and

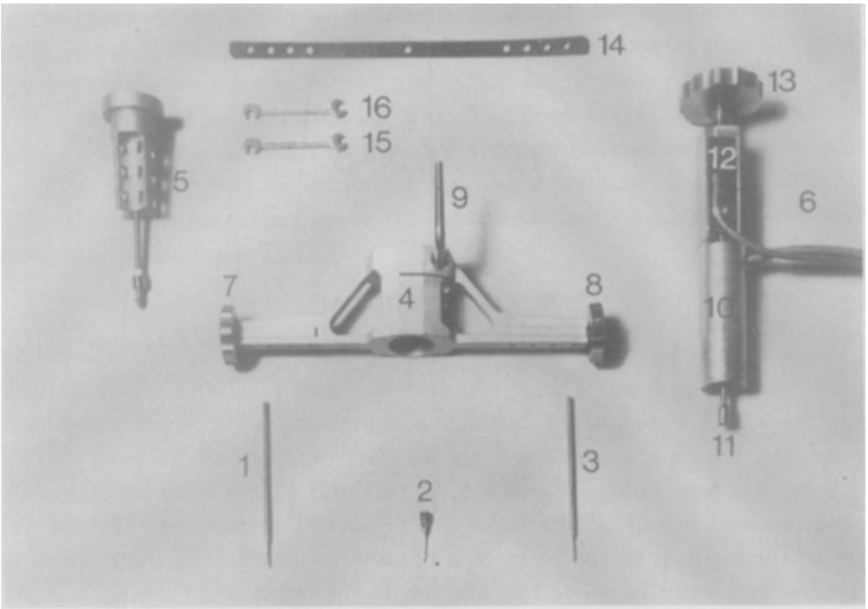


FIGURE 6 *The measuring instrument.*

1, 3	*Outer pins*
2	*Middle pin*
4	*Measuring beam*
5	*Template for positioning of middle pin*
6	*Measuring cylinder with built-in position transducer*
7, 8	*Wheels for locking outer pins*
9	*Lever for locking measuring cylinder*
10	*Position transducer, built-in*
11, 13	*Wheel 13 for displacering shaft 11 on force transducer; the shaft passes through the position transducer to apply force to the bone*
12	*Force transducer*
14	*Template for ensuring correct position of local anaesthetic*
15, 16	*Spanners for nut on shaft 11*

inside which are four strain gauges, attached at different distances from each other. When loaded, the cylinder is deformed so that its cross section is elliptical and the change in the radius of curvature of the cylinder is registered by the strain gauges, which form a 4-armed measuring bridge. The force transducer is connected to a transducer amplifier indicator, (Bofors, Type BKI-1), which contains circuits for measurement, and for balancing and calibration of the transducer bridge, and an amplifier. The deviation from linearity and hysteresis are less than 0.2 per cent and the calibration accuracy ±0.5 per cent.

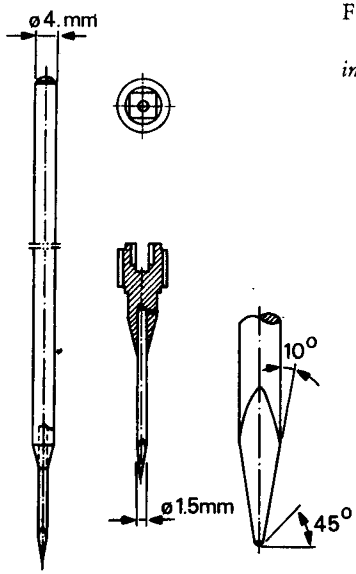


FIGURE 7 *Left.—Outer pin. Centre.—Middle pin. Right.—Enlargement of the pin tip showing conical shape of end.*

The signals produced by the position and force transducers are led to an X-Y recorder (Moseley, Model 7035 A), which registers a load-deflection diagram; accuracy ±0.2 per cent; linearity ±0.1 per cent. The *Y* coordinate denotes the displacement of the middle pin in relation to the position transducer, and the *X* coordinate the compressive or tensile force acting on the pin. The instrument can be set so that a full-scale reading on the Y-axis corresponds to a displacement of 0.5, 1, 2, 5 or 10 mm, and the full-scale deflection on the *X*-axis to a force of 1, 5 or 10 kgf. For most of the measurements in the investigation the full-scale deflection represented 0.5 mm and 5 kgf, respectively.

PRINCIPLES OF MEASUREMENT

The tibia is loaded by fixing the measuring beam to the bone with the outer pins and applying a bending force between the beam and the bone by means of the screw which moves the middle pin towards or away from the bone.

The method of measurement is based on the principle governing the bending of two beams connected at the ends and subjected to a bending force applied at the midpoint. In the present case one of the beams consists of the tibia and the other of the measuring beam. By regarding the tibia as a beam in the composite mode of loading it is possible to apply the

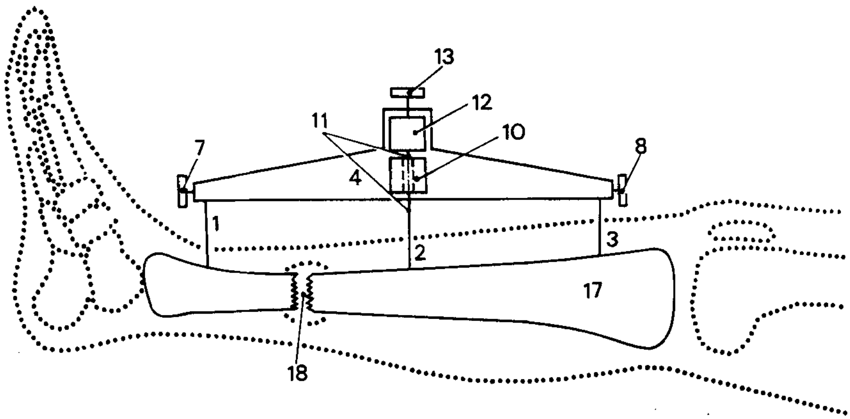


FIGURE 8 *The instrument mounted on a fractured tibia. See legend to figure 6. 17 Tibia 18 Fracture.*

principle pertaining to the deflection of a straight uniform beam simply supported and loaded at its midpoint.

The load is applied by turning the wheel *13* (Fig. 8) so that the shaft *11* with the middle pin *2* presses against the bone *17*. The tibia and the measuring beam are thus bent with their convexities in opposite directions with a compressive force *P*; the reaction *P/2* produces a tensile stress in each outer pin. When the measuring beam and the bone are deflected in this way the outer pins, which are locked in the beam and fixed in the bone, will bend with their convexities towards the middle pin; when the shaft *11* is subjected to a tensile force, the opposite loading conditions will obtain, so that the tibia and the measuring beam are bent towards each other. The deflection is thus measured in both positive and negative directions.

CALIBRATION

The X-Y recorder was connected to the position and force transducers and the instruments were calibrated and the linearity was checked over various measuring ranges. On several occasions during the course of the investigation the force transducer was calibrated against a load of 5 kgf and the reading on the X-Y recorder was checked. The position transducer was checked with the aid of pieces of sheet metal, whose dimensions had been determined with a micrometer screw gauge. No corrections were required during the course of this investigation. The instrument was also calibrated in a simpler way—against a steel bar to which the measuring

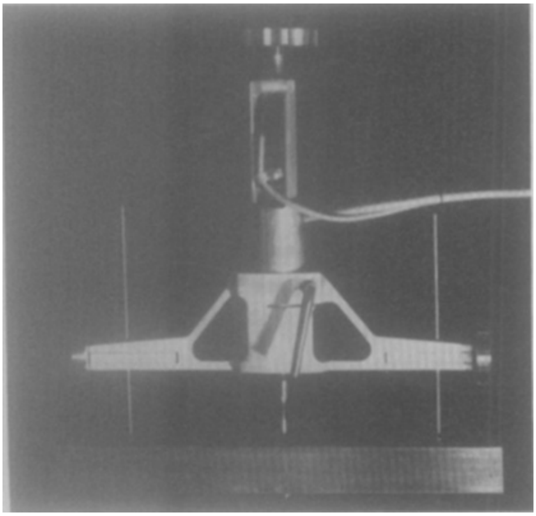


FIGURE 9 *The measuring instrument bolted to a steel beam used for checking that the deflection at a bending force of 5 kgf lies within the pre-determined limits.*

beam was bolted (Fig. 9). The same bar was used for measuring the deformation of the measuring beam, as described below.

MEASURING ERROR

The position transducer indicates the total deflection of the beam and the object when a load is applied. The deflection of the beam was determined by fixing it to a steel bar measuring 32 by 32 by 300 mm made of 1.5 mm sheet steel. It was fixed with pins, the tips of which were soldered to bolts. The outer bolts were connected to the bar at an interval of 24 and 21 cm.

The deflection of the steel bar produced by a bending force of 5 kgf exerted by the middle pin is theoretically 3.8 µm for 24 cm and 2.5 µm for 21 cm. The deflection of the steel bar and the measuring beam with 24 cm between the bolts was 24.2 µm (range 22—27; $n=11$) for 5 kgf, and 48.5 µm (range 44—52; $n=8$) for 10 kgf. With 21 cm between the bolts the deflection was 14.9 µm (range 13—17; $n=11$) for 5 kgf, and 29.7 µm (range 27—32; $n=11$) for a force of 10 kgf.

The total mutual displacement of the tibia and the beam recorded by the position transducer includes the local movement of the cortex around the needles. As a compressive force on the outer pins will result in a tensile force on the middle pin, and *vice versa*, the error due to this local deformation of the bone consists of the displacement of the middle pin and half the sum of the displacement of the outer pins.

The deformation of the actual pins also constitutes a source of error, but in measurements on bone this does not amount to as much as 1 per cent of the total deformation.

The sum of the deformation of the measuring beam, of the cortex around the pins and of the pins themselves was determined as follows. Twelve tibia specimens were sawn through their length, ground plane on the surface that was to be placed against the steel bar, and fixed to this bar with six bolts. Each specimen was tested twice at the two pin distances 24 and 21 cm. ("Pin distance" refers to the distance between the outer pins.) The deflection in the negative and positive directions produced by a load of 5 kgf was determined five times in each direction. There was no difference in the deflections in the two directions; the mean for the pin distance of 24 cm was 42.9 μm (range 32—63) and for 21 cm it was 31.3 μm (range 22—41). These values *minus* the deflection of the steel bar for the respective pin distances gives the correction to be subtracted from the observed measured displacement in an examination of the deflection of the bone. For a force of 5 kgf the corrections were 39 μm and 29 μm with 24 and 21 cm between the outer pins.

MEASURING TECHNIQUE

The instrument is intended for use in diaphyseal fractures of the long bones. The tibia was chosen for the investigation because fractures of this bone are common and not infrequently display delayed union.

The level at which the instrument shall be placed on the tibia is decided by the position of the fracture. The deflection is greater the closer the middle pin is to the fracture; this distance should not be less than 2 cm, because during the union the cortex becomes softer and does not provide so firm a hold for the pin. The mean distance from the centre of the fracture to the pin was 60 mm. For extreme distal or proximal sites of fracture an outer pin may have to be placed in the malleolar and condylar part of the tibia; here the cortex is thin and there is a risk that the pin will not be held firmly enough. There is also a risk that when it is being inserted the cortex may easily be penetrated, with consequent weakening of the hold.

In such cases, it would be wiser to use a distance of 21 cm or less, but for the sake of uniformity in the assessment of the results 24 cm was used in all but two cases, and here the distance was 21 cm.

When the level at which the instrument is to be placed has been determined it is marked on the fractured leg, and also on the sound leg. The level of the middle pin was measured as the distance from the medial articular space of the knee or the medial malleolus. A local anaesthetic is administered at the intended sites in the skin and down towards the periosteum. The correct location of the anaesthetic is ensured by using the template *14* (Fig. 6), which consists of a stainless steel strip with holes corresponding to the positions of the middle pin and the pin distances on the measuring beam.

The outer pins are inserted in the holes in the beam and locked by gentle tightening of the wheels *7* and *8*. On the shaft of the template *5* the middle pin is fixed and the template inserted in its cylindrical holder. The pins are placed perpendicular to the medial surface on the tibia. With a light tap of a hammer the pins are inserted through the skin and periosteum down to the bone surface. The wheels *7* and *8* are then loosened and while the beam is held in one hand the pins are hammered about 2 mm into the cortex. They must be securely held by the bone but not penetrate to the marrow space. The outer pins are inserted first, followed by the middle one. The outer pins are locked to the measuring beam by tightening the wheels *7* and *8*. The nut on the shaft passing through the template is unscrewed from the middle pin, and the template with the shaft removed. After the position and force transducers have been set at zero, the measuring cylinder is locked to the middle pin by the nut on the shaft with the aid of spanners *15* and *16*. When the measuring cylinder is in place it is locked to the beam by tightening the lever *9*. As the measuring cylinder cannot be sterilized, from now onwards the sterility in the investigation is broken and the greatest care must be observed to avoid infection of the small lesions due to the

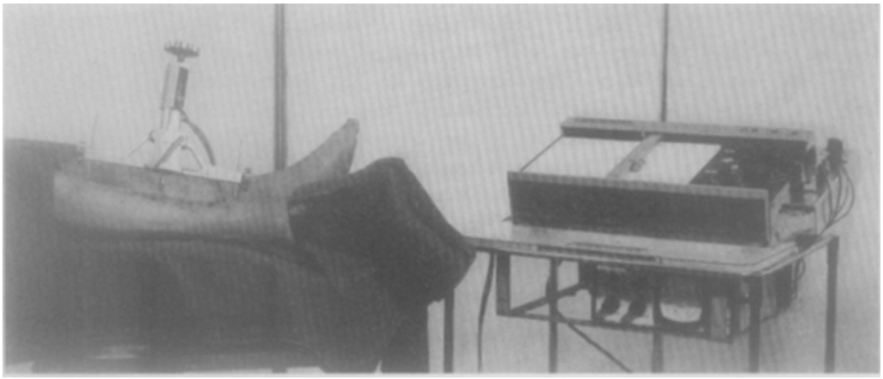


FIGURE 10 *The measuring instrument applied to the bone in the living patient for measurement of deflection. On the right is the X—Y recorder, and below it the transducer, amplifier, and bridge.*

pins. The legs rest on a foam rubber cushion, usually slightly everted so that the instrument is perpendicular and rests on the bone, or else it is steadied with one hand (Fig. 10).

The instrument is now ready for measurement of the load on, and deflection of, the tibia. When the wheel of the cylinder is turned clockwise the middle pin moves down to bear on to the bone and the X-Y recorder registers a curve in the negative direction of x. The wheel is turned until a full reading corresponding to a compressive force of 1, 5 or 10 kgf is obtained on the middle pin, which produces a deflection in the negative direction. By turning the wheel counterclockwise the compressive force on the middle pin is first decreased to zero and then a tensile force is produced until a full-scale deflection is obtained in the positive direction of x. Examples of the curves recorded for the tibia specimens are shown in figure 11.

It is important to ensure that the pins do not loosen during the measurements. To avoid disturbing the outer pins while the middle pin is being hammered in they should not be locked until afterwards. If a pin penetrates the cortex it may slide in relation to the bone. Curve C in figure 12 shows how a change on the position of the middle pin by turning the wheel on the measuring cylinder results in an increase in the reading from the position

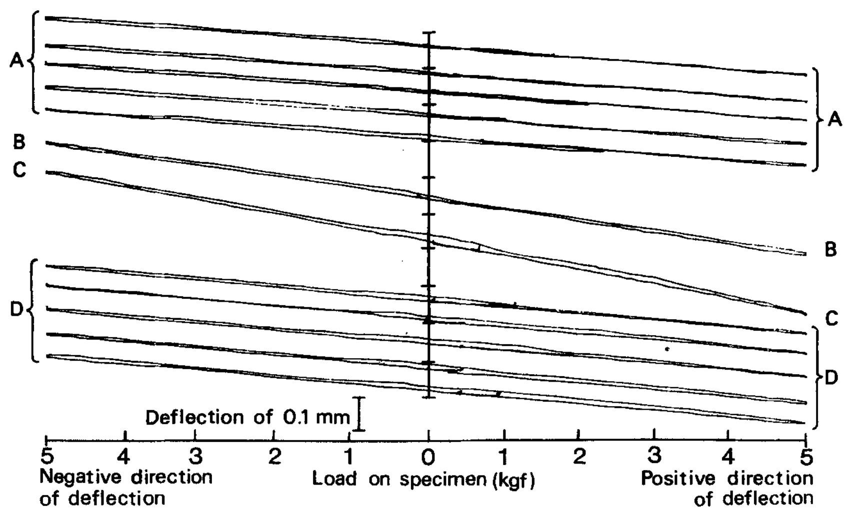


FIGURE 11 *Load–deflection curve for tibia specimen.*
A, B.—Pin distance of 21 cm; load 5 and 10 kgf, respectively.
C, D.—Pin distance of 24 cm; load 10 and 5 kgf, respectively.

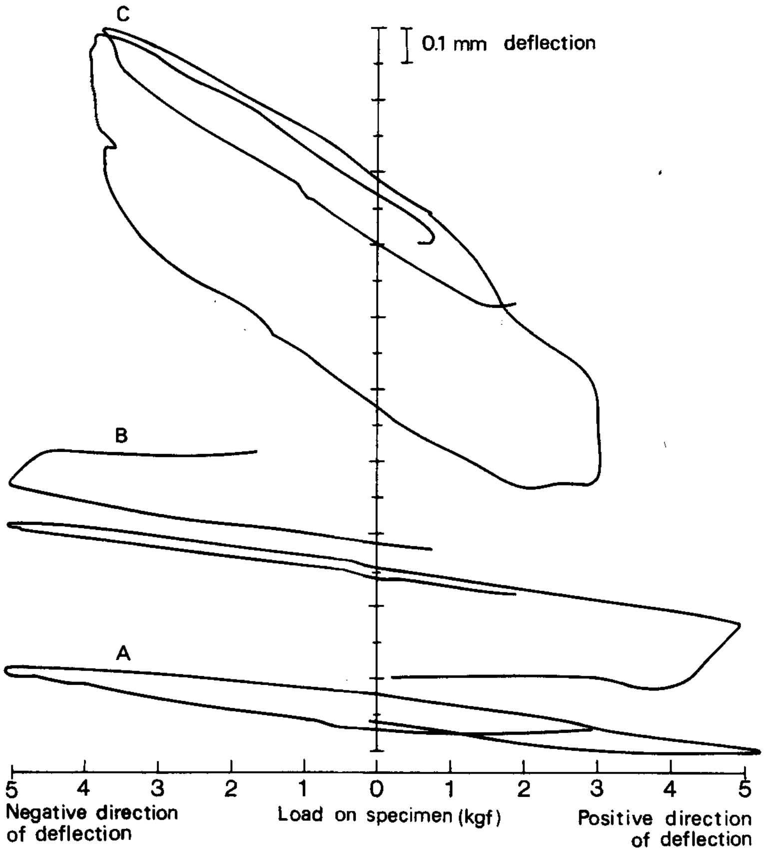


FIGURE 12 *Load-deflection curve for vital tibia.*
A.—*Curve showing hysteresis due to high friction of measuring cylinder shaft against position transducer owing to oblique middle pin.*
B.—*Because the pin had loosened the curves tend towards the axis of y after an initial short increase in deflection.*
C.—*Slipping of the pin produced a large displacement parallel to the axis of y, with no change in applied force.*

transducer—that is along the *y*-axis of the recorder—but no increase in the force already applied, which remains constant at a value of *x* corresponding to the force required to overcome the frictional resistance of the pin in the bone.

When a pin loosens, the stress in the beam is relieved and the beam recovers its original shape, the resulting displacement producing an increase in deflection. The force on the middle pin is also relieved and *x* again becomes zero. Curves B in figure 12 recorded when a pin loosened show how the force diminishes and the curves run parallel with the x-axis towards zero.

When the middle pin is being inserted it may bend or be guided along an oblique path in the bone substance. This is indicated by difficulty in getting the measuring cylinder into position on the middle pin, it being necessary to move its head slightly to the side before the shaft of the measuring cylinder can be screwed fast. This, however, will result in too large a deflection, since the position transducer also registers any deformation of the pin. The error will be greatest if the deviation of the middle pin from the plane of deflection of the instrument occurs in the transverse plane of the tibia, because then a torque will be produced that will also tend to bend the outer pins in a transverse plane. While a middle pin is inclined in the bending plane of the tibia it will not tend to rotate the bone. This situation is reflected in a curve as an increase in hysteresis. It is due to the fact that the axis of the measuring cylinder bears against the housing of the position transducer, and the force required to overcome the resulting friction is recorded without simultaneous change in the position of the axis; this is illustrated by curve A in figure 12.

When the measurement is completed the nut holding the middle pin is loosened and the lever *9* and the measuring cylinder are removed. The outer pins are unlocked and the beam lifted off.

The measuring procedure is then repeated on the sound leg and radiographs are taken with all the pins in place in both tibias.

RADIOGRAPHIC EXAMINATION

After the measurements radiographs are taken with the exposure data kept constant for each patient. On the radiographs an estimate of the distance between the pins and the fracture is obtained and the dimensions of the tibia are measured for the determination of the moment of inertia; they also serve for the usual examination of the fracture. The radiographs are taken in two projections, one perpendicular to the pins and in the plane of the medial surface of the tibia, and the other parallel to the pins. The central ray is centred on the middle pin. As the distance between the outer pins is known a reliable enlargement factor can be obtained by measuring the distance on the radiograph. By means of this factor the dimensions of the tibia and the distance of the pins from the fracture can be accurately determined.

The magnitude of the deflection within the fracture is strongly dependent on its distance from the nearer outer pin. On films of the fractured tibia the distance from the middle pin to the line of fracture is measured in the projection that gives a side view of the pins. Two distances are determined, namely, that from the middle pin to the fracture line on the same side as the pin is inserted, and that to the fracture line in the cortex of the opposite side. The distance from the centre of the fracture to the level of the middle pin is calculated as half the sum of the above two distances multiplied by the enlargement factor. The distance so obtained subtracted from the distance between the middle pin and the relevant outer pin gives the distance from the nearer outer pin to the fracture—l in eqn. [11].

The moment of inertia for the narrowest part of the sound tibia shaft is calculated. On films taken with the projection giving a lateral view of the pins the narrowest part of the tibia is found, and as far distally as possible within this part measurements are made of the distance d, the thickness of the tibia to the medial surface. The diameter a of the marrow space at the same level is also determined. The distance of the middle pin from this level in this projection is marked off from the middle pin on the radiograph exposed with the perpendicular projection, and here the width b of the tibia is determined.

The dimensions so obtained are corrected with respect to the enlargement factor and entered in eqn. [8]. $I_{appr} = 0.042\ bd^3 - 0.049\ a^4$, which gives an approximate value of the moment of inertia of the tibia at it narrowest part.

V. Measurements on autopsy tibias

One of the objects of the experiments on autopsy specimens was to determine the error contained by the recorded value of the deflection of the tibia in a single observation. Another was to check the agreement between the values of the moment of inertia determined by (*i*) numerical integration, (*ii*) with an integrator and (*iii*) by application of eqn. [8] for I_{appr}.

MATERIAL

The pairs of tibia specimens were obtained at autopsies performed at the National Forensic Station from persons that had died through trauma or suicide and that, so far as was known, had not had any disease that may have affected the bones. The mean age of these subjects was 32 years (range 22—44). Other data are presented in table 1.

The fresh bone specimens were preserved in Ringer's solution. The deflection measurements were usually performed on the day the specimens were prepared, and never later than the following day. Pending some later examination they were stored at —20°C in polythene bags containing Ringer's solution. Throughout the work on the specimens the moisture con-

TABLE 1 *Autopsy series.*

		Age	Sex	Cause of death			Age	Sex	Cause of death
Subject	1	26	M	Suicide	*Subject*	12	31	F	Suicide
	2	32	F	„		13	22	M	Subarachnoid haemorrhage
	3	26	F	„					
	4	39	M	Cranial fracture		14	44	M	Suicide
	5	37	M	Suicide		15	34	M	„
	6	24	M	„		16	35	M	Epileptic convulsions
	7	30	M	„		17	39	M	Suicide
	8	26	M	„		18	26	F	Thoracic and abdominal traumata
	9	39	M	„					
	10	30	M	„		19	31	M	Suicide
	11	39	F	Abdominal trauma		20	29	M	„

tent was retained and for only short intervals were they exposed to the room atmosphere. To reduce evaporation from the bone a thin layer of the investing soft tissues was left in place as long as possible. The actual measurement was performed at room temperature. The directions given by Sedlin (1965) and Sedlin & Hirsch (1966) for preserving the physical properties were followed.

DETERMINATION OF DEFLECTION

The length of the tibia was taken as the distance between the distal articular surface of the bone and the middle portion of the articular surface of the medial condyle. To obtain the midpoint of the tibia, one half this distance was then marked off from these articular surfaces on the medial surface. The lengths of the specimens are given in table 2. The middle pin of the instrument was anchored at a point half way along and across the tibia, perpendicular to the medial surface (Fig. 13). One measurement was

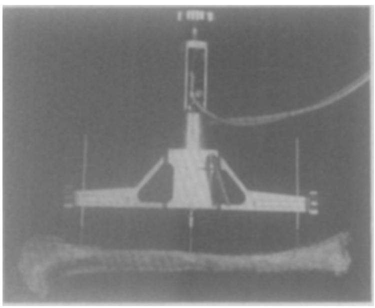


FIGURE 13 *Measuring instrument applied to a bone specimen.*

made with each of the pin intervals 24 and 21 cm. The middle pin was subjected to alternately tensile and compressive forces of up to 5 kgf—both away from and towards the measuring beam (positive and negative directions). No difference in the deflection for these two directions was recorded. Each measurement was repeated five times in both directions and the mean deflection for the 10 measurements for a load of 5 kgf was determined. The recorded curves for a tibia are presented in figure 11.

It was impossible to determine the error of deflection for a single specimen by replicate measurements because for the second measurement the experimental conditions are different. For example, any alteration in the fixation of the pins would result in a change in the plane of bending and the moment of inertia. The error of deflection for a single observation was therefore determined in two ways, namely by measuring the deflection at pin distances of 24 and 21 cm and by comparing the deflections on pairs of

TABLE 2 *Results of measurements on the autopsy specimens. Length of tibia; moments of inertia obtained from equation* [8] $I_{min\ appr}$, with integrator $I_{min\ i}$ *and by a numerical integration* $I_{min\ n}$; *and the corrected deflection of the tibia for a load of 5 kgf at pin distances of 24 and 21 cm.*

Specimen		Length of tibia	Moment of inertia (cm^4) $I_{min\ appr}$	$I_{min\ i}$	$I_{min\ i\ corr}$	$I_{min\ n}$	$(I_{min\ i\ corr} + I_{min\ n})/2$	Deflection (μm) 24	21
Right	1	400	1.3267	1.3139	1.2186	1.3044	1.2615	34	—
	2	358	0.5165	0.4940	0.4685	0.4865	0.4775	94	68
	3	356	0.8966	0.8212	0.7713	0.7695	0.7704	70	43
	4	397	0.8017	0.8653	0.8153	0.8529	0.8341	60	—
	5	340	1.0967	1.0340	0.9628	1.0090	0.9859	52	39
	6	378	1.4947	1.3512	1.4192	1.3984	1.4088	56	41
	7	420	0.8448	0.9037	0.8349	0.8512	0.8431	74	48
	8	360	0.5730	0.5476	0.5747	0.5932	0.5840	91	56
	9	376	1.0729	1.0270	0.9570	0.9249	0.9410	59	44
	10	382	1.1405	1.1360	1.0761	1.0738	1.0750	53	37
	11	337	0.4146	0.4297	0.4537	0.4302	0.4420	125	85
	12	327	0.7640	0.7002	0.6554	0.6335	0.6445	79	56
	13	393	1.0874	1.1910	1.1187	1.1176	1.1182	49	36
	14	360	0.9100	0.7701	0.7256	0.7250	0.7253	70	46
	15	350	0.6420	0.6321	0.5937	0.5739	0.5838	84	66
	16	384	1.5001	1.3203	1.2440	1.2715	1.2578	57	33
	17	388	1.2000	1.1325	1.0670	1.0920	1.0795	61	35
	18	304	0.4866	0.5568	0.5831	0.5910	0.5871	107	66
	19	376	0.8534	0.7567	0.8104	0.8405	0.8255	73	62
	20	355	0.9981	1.0494	1.1193	1.1080	1.1137	61	53
Left	1	398	1.3344	1.2498	1.3278	1.3665	1.3472	31	—
	2	358	0.4260	0.4121	0.4295	0.4414	0.4355	115	78
	3	356	0.9780	0.9076	0.9496	0.9317	0.9407	59	37
	4	396	0.7935	0.8268	0.8651	0.8570	0.8611	61	43
	5	340	0.9276	0.8836	0.9281	0.8445	0.8863	63	52
	6	376	1.4039	1.3575	1.4258	1.3908	1.4083	50	41
	7	419	0.7460	0.8185	0.8590	0.8221	0.8406	67	46
	8	362	0.5250	0.5227	0.5486	0.5434	0.5460	79	57
	9	380	1.1370	0.9261	0.9660	1.0476	1.0068	88	43
	10	379	1.0340	1.0568	1.1160	1.1363	1.1262	56	41
	11	338	0.6044	0.6081	0.6330	0.6088	0.6209	91	65
	12	330	0.6537	0.6156	0.6421	0.6034	0.6228	88	64
	13	393	1.1122	1.2026	1.1321	1.3197	1.2259	57	31
	14	360	1.0665	0.9060	0.9490	0.8465	0.8978	64	46
	15	350	0.5909	0.5678	0.5350	0.6141	0.5746	98	63
	16	379	1.5843	1.4733	1.3882	1.4011	1.3947	51	30
	17	387	1.0812	1.0505	0.9898	1.1108	1.0503	56	46
	18	314	0.4666	0.5012	0.5218	0.5320	0.5269	134	78
	19	371	0.8568	0.7564	0.7914	0.7914	0.7914	74	45
	20	355	1.0473	1.0491	1.0943	1.0905	1.0924	73	49

tibias. In the former method it should be possible to keep the same plane of bending.

No measurements were made for the 21-cm distance for either tibia of subject no. 1, nor for the right tibia of subject no. 4. The marked hysteresis displayed by the curves for subject no. 9, left tibia, 24-cm pin distance, is due to an oblique position of the middle pin, and the value for this curve was therefore discarded.

The values of δ_{24} and δ_{21} obtained after correction for the error of the method (instrument deformation, page 24) are given in table 2.

DETERMINATION OF THE MOMENT OF INERTIA

The magnitude of the deflection is dependent on the moment of inertia I and the modulus of elasticity E of the bone, and to enable a comparison of the deflection of different bones to be made the moment of inertia of the cross section was determined. This was done in three ways: (*i*) numerical integration, (*ii*) with an integrator and (*iii*) by using eqn. [8]. The transverse section of the tibia was chosen at the level where the thickness of the bone d was smallest and the moment of inertia of the section therefore lowest (page 13).

To obtain as accurate a comparison as possible of the deflection of different tibias the first two methods were used for replicate determinations of the moment of inertia.

(i) Numerical integration

The moment of inertia was determined numerically ($I_{min\ n}$) from ink impressions of the cross section of tibia obtained on graph paper. The cross section had previously been ground plane on water-resistent sandpaper under water and the fat in the marrow space and the looser spongy bone scraped away. The section was placed so that the side included in the medial surface was parallel to one of the set lines of the graph paper. For each section the moment of inertia about the central axis was calculated twice; this was done by summing the products of the square of the distance of each millimetre square in the section area from the central axis, and adding to this total the sum of the moment of inertia of each millimetre square about its own central axis. The mean of the two values of the moment ($I_{min\ n}$) is given in table 2.

(ii) Integrator

The cross section of the tibia was photographed with a millimetre scale placed in the same plane. The films were copied and enlarged by a factor of 10 and the moment of inertia was ($I_{min\ n}$) determined with an Amsler integrator Type 2002. The method error for the integrator is ± 1 per cent for the measurement of area and ± 2 per cent for moment of inertia. The sections were placed so that the side included in the medial surface was parallel with the integrator ruler. Two determinations of $I_{min\ i}$ were made on each section; their mean and the corrected values of $I_{min\ i}$, designated ($I_{min\ i\ corr}$), are given in table 2. The correction takes account of non-uniform enlargement.

(iii) Formula

On the photographic enlargements of the sections produced for method (*ii*) measurements were made of the depth of the tibia (d), the width of the section (b) and the diameter of the marrow space (a), as these dimensions would be obtained by radiographic projection by the technique mentioned on page 29. These values were inserted in eqn. [8] for the approximate moment of inertia: $I_{min\ appr} = 0.042\ bd^3 - 0.049\ a^4$.

RESULTS

Deflection

As is evident from eqn [9], if the tibia is regarded as a simply supported beam loaded at its midpoint, the corrected deflections, δ_{24} and δ_{21}, are related as the cubes of the respective outer pin distances multiplied by the constants 1/67 and 1/66, respectively. The theoretical ratio of δ_{24} and δ_{21} is then determined thus: $(24.1^3/67)/(21.0^3/66)$, or 1.49. By dividing the value of δ_{24} by that of δ_{21} from each specimen, an experimental value for the ratio was obtained. A preliminary analysis disclosed no difference between the quotients for the right and the left tibias. The mean for all 36 specimens was then 1.46. The theoretical ratio lay within the 99 per cent confidence interval for the experimental ratio.

By multiplying δ_{21} by the theoretical ratio 1.49, a value is obtained that can be compared with δ_{24}. The series of values δ_{24} and 1.49 δ_{21} were submitted to analysis of variance to determine the experimental error incurred in the deflection of the tibia (Table 3). As it was only the values for one and the same bone that were compared, determinations on both right and left tibias could be used in the determination of the measurement error. In the analysis of variance the two series of deflection values from the tibias were treated as a two-factor experiment, the two factors being the specimen and the pin distance (*Snedecor & Cochran*, Chapter 12). However, the sum of the squares of interaction cannot be separated from the sum of the squares of the error (Table 3).

TABLE 3 *Analysis of variance for the corrected deflections of tibia specimens. The values of δ_{24} and 1.49 δ_{21} are analysed. The values of δ_{24} and δ_{21} are given in table 2.*

	Degrees of freedom	Mean square	F ratio	P
Source of variation				
Specimen	35	866		
Pin distance	1	65	$F_{1;35}=1.81$	>0.05
Error and interaction[a]	35	36		

[a]Relative error for a single measurement: $100(36)^{1/2}/75=\pm 8$ per cent

There was no significant difference between δ_{24} and 1.49 δ_{21}. The relative experimental error for a single determination of the deflection was thus ± 8 per cent.

The experimental error incurred in measuring the deflection was also determined in another way. The modulus of elasticity was then assumed to be the same for the two tibias of a subject. The relative error of the modulus of elasticity is equal to the square root of the sum of the squares of the relative errors of the moment of inertia and the deflection, as the modulus of elasticity is related to the product of the deflection and the moment of inertia, according to eqn. [2]. Thus, the error in the corrected observed deflection of the tibia can be calculated if the relative errors of the determinations of the modulus of elasticity and the moment of inertia are known. The apparent modulus of elasticity "E_{24}" and "E_{21}" was calculated from the mean value of $I_{min\ i\ corr}$ and $I_{min\ n}$ for each specimen, and the corresponding values of the deflections δ_{24} and δ_{21}. In the analysis of variance for the determination of the experimental error, the same model as in the previous experiment was used. The two factors were the subject and the side. The interaction between subject and side could not be isolated from the error of measurement. The analyses of variance presented in table 4 show no significant difference between the right and left sides for either "E_{24}" or "E_{21}". The means for "E_{24}" and "E_{21}" were 2.6×10^3 and 2.4×10^3 kgf/mm^2, respectively. The corresponding experimental relative errors were ± 8 and ± 9 per cent.

The experimental relative error in determining the moment of inertia was ± 4 per cent, as reported below. For the mean of two determinations of $I_{min\ i\ corr}$ and $I_{min\ n}$, as used in the calculation of the apparent modulus of elasticity, the corresponding relative error will be ± 3 per cent. Thus, the relative error for a single measurement in δ_{24} was obtained as ± 7 per cent and in δ_{21} as ± 8 per cent.

TABLE 4 *Analysis of variance for the apparent modulus of elasticity of the tibia specimens. Separate analyses were performed for the pin distances 24 and 21 cm.*

	Degrees of freedom	Mean square $\times 10^6$	F ratio	P
	PIN DISTANCE 24 cm			
Source of variation				
Subject	18	3048		
Side	1	679	$F_{1;18}=1.61$	>0.05
Error and interaction[a]	18	423		
	PIN DISTANCE 21 cm			
Subject	17	2385		
Side	1	455	$F_{1;17}=0.88$	>0.05
Error and interaction[b]	17	519		

[a]Relative error for a single determination: $100(423\times 10^6)^{1/2}/258{,}000=\pm 8$ per cent

[b] ,, ,, ,, ,, ,, ,, : $100(519\times 10^6)^{1/2}/241{,}000=\pm 9$ per cent

Thus, the relative error of the deflection of the tibia for a single determination was ± 8 per cent for both methods of determining the experimental error.

Moment of inertia

The values obtained for the moment of inertia from eqn [8] $I_{min\ appr}$ and from the determination with the integrator $I_{min\ i}$ were compared (Table 2). As they were determined from the same enlarged photographs, no refined correction for the small difference in the enlargement factor between different pictures is necessary. By analysis of variance, the factors considered to influence the measurements were examined. The main sources of variation were side (I), method (I) and subject (III). There was also the interaction between side and method (I &II), side and subject (I & III), and method and subject (II & III) and the three-factor interaction (I, II & III) could not be separated from the experimental error. Statistical models are given in *Snedecor & Cochran*, Chapter 12.

The analysis of variance (Table 5) shows that there is no significant difference between sides or methods and there is a highly significant lack of additivity between side and subject and also between method and subject. The estimated relative error for a single determination of $I_{min\ appr}$ is ± 3 per cent.

TABLE 5 *Analysis of variance for the minimum moment of inertia of the narrowest tibia section derived from values of $I_{min\ appr}$ and $I_{min\ i}$ (Table 2).*

	Degrees of freedom	Mean square	F ratio	P
Source of variation				
Side, I	1	0.00437	$F_{1;19} = 0.55$	>0.05
Method, II	1	0.02000	$F_{1;19} = 4.00$	>0.05
Subject, III	19	0.35981		
Interaction, I & II	1	0.00001		
„ I & III	19	0.00801	$F_{19;19} = 10.97$	<0.001
„ I & III	19	0.00500	$F_{19;19} = 6.85$	<0.001
Error and interaction[a] I, II & III	19	0.00073		

[a]Relative error for a single determination: $100(0.00073)^{1/2}/0.91 = \pm 3$ per cent
Means: $I_{min\ appr} = 0.92$; $I_{min\ i} = 0.89$ cm^4.

In exactly the same manner the values of the moments of inertia arrived at by numerical integration ($I_{min\ n}$) and the corrected values obtained by the integrator method ($I_{min\ i\ corr}$) were analysed. These two series of values are presented in table 2, and the results of the analysis of variance are given in table 6. There was no significant difference between sides or between methods. A highly significant lack of additivity between side and subject was obtained but in this experiment there was no significant lack of additivity between method and subject.

The approximation method for determining the moment of inertia contains an error the magnitude of which increases with the disparity between

TABLE 6 *Analysis of variance for the minimum moment of inertia of the narrowest tibia section derived from values of $I_{min\ i\ corr}$ and $I_{min\ n}$ (Table 2).*

	Degrees of freedom	Mean square	F ratio	P
Source of variation				
Side, I	1	0.02034	$F_{1;19} = 2.85$	>0.05
Method, II	1	0.00185	$F_{1;19} = 1.45$	>0.05
Subject, III	19	0.32696		
Interaction, I & II	1	0.00001		
„ I & III	19	0.00713	$F_{19;19} = 5.17$	<0.001
„ I & III	19	0.00128	$F_{19;19} = 0.93$	>0.05
Error and interaction[a] I, II & III	19	0.00138		

[a]Relative error for a single determination: $100(0.00138)^{1/2}/0.89 = \pm 4$ per cent
Means: $I_{min\ i\ corr} = 0.89$; $I_{min\ n} = 0.90$ cm^4

the configuration of the cross section and that of the standardized contour. This explains why the mean square of the interaction between method and subject is larger in the analysis of $I_{min\ appr}$ and $I_{min\ i}$ than in that of $I_{min\ i\ corr}$ and $I_{min\ n}$.

DISCUSSION

Factors affecting the deflection

The maximum deflection of the tibia is obtained if the bending force acts in the bending plane for the minimum moment of inertia of the section. Since the directions of the principal central axes are unknown at the time of the measurement, the recorded deflection will include the error due to the instrument being placed perpendicular to the medial surface of the tibia. If the direction of the load makes an angle β with the plane for maximum deflection of the tibia the bending will occur in a plane making an angle of φ with the plane for maximum deflection, where the relationship between these angles is given by eqn. [6]. The deviation of the recorded deflection from the maximum is obtained from eqn. [7]. By way of example, consider the case where $\beta=15°$, $I_{max}=1.5$ and $I_{min}=1.0$; then $\varphi=10.1°$. These values entered in the formula give $v_{obs}=0.977\ v_{max}$. The recorded deflection of the tibia is thus 2.3 per cent smaller than the maximum deflection as a result of the angle between the plane perpendicular to the plane of the medial surface and the plane for maximum deflection of the tibia. As this angle will probably not exceed 15°, the given example will illustrate the influence of the angle between the planes on the error of the deflection in a most unfavourable case.

To see how the angle between the plane perpendicular to the medial surface and the bending plane for maximum deflection of tibia varies, the directions of the principal central axis of seven sections of widely different shapes were examined. The direction of one of the principal central axes lies in the plane for maximum deflection of the tibia. By means of the integrator the principal moment of inertia and the positions of the principal central axes were determined. The greatest value of the required angle was 9.3°, which implies an increase of 2.2 per cent in the corresponding moment of inertia from the minimum moment. The results from the other sections are given in table 7. The highest value for the ratio I_{max}/I_{min} was 2.6.

According to the determinations performed on autopsy specimens, the principal central axes in different sections are practically parallel (Fig. 3), and the tibia would thus not itself produce a rotation while being bent.

TABLE 7 *Angle β between chosen plane of bending for the instrument and the bending plane for the minimum moment of inertia. Increase in the moment of inertia corresponding to observed angle β. Ratio I_{max}/I_{min}.*

Subject	1	2	3	5	13	14	18
Angle β between chosen and principal central axis; degrees	7.3	—0.5	1.5	9.3	7.8	—1.5	<0.1
$I\beta - I_{min}$ as a percentage of I_{min}	1.4	<0.1	<0.1	2.2	0.5	0.2	<0.1
I_{max}/I_{min}	1.85	1.29	1.54	1.96	1.28	2.56	1.79

The deflection curve of a particular tibia is unique. As pointed out on page 15, this curve is dependent on the variation of the minimum moment of inertia for the section along the length of tibia. If the weakest section were to be located at the level of the middle pin, this would be the site of the greatest possible deflection of the tibia.

As the centre of torsion and the centroid of the tibia section do not coincide, the deflection of the tibia will be combined with a rotation. The calculations show that in the least favourable case the error in the recorded deflection due to this rotation does not amount to more than +2 per cent.

Factors governing the moment of inertia

The moment of inertia is difficult to measure in spongy bone, but the narrowest part of the tibia contains only a small amount of such substance. It is difficult to draw a boundary between the cortical and the spongy bone, and the border has been chosen where the cavities of the spongy bone merge with the marrow space. Proximally and distally of the narrowest part of the tibia the spongy bone occupies an increasingly large proportion of the section, and it is difficult to judge its share of the moment of inertia. For other than the narrowest part, it is thus mainly the moment of inertia of the cortical bone that was determined. This means that the value for the sections with a greater proportion of spongy bone is on the low side.

Compact bone varies in porosity and there are divergent opinions on the effect of this structural factor on the physical properties of the bone (Evans, 1958; Smith & Walmsley, 1958; Currey, 1959; Vose, 1961; Sedlin & Hirsch, 1966; Blaimont, 1968; Evans & Bang, 1968). The fact that the subjects from which the bone specimens were obtained were not above 44 years of age would seem to rule out the possibility of osteoporosis and the specimens probably dit not differ essentially in porosity.

The errors in the determination of the deflection and the moment of inertia will evidently affect the observed modulus of elasticity. A large error is due to the constant in eqn. [9]. An accurate determination of the modulus of elasticity would need a specific constant for each bone because of the individual structure of the tibia. It was to some extent surprising that there was not a greater difference in the constant for the five specimens for which it was calculated (standard deviation 5 per cent). With the mean of 1/67 for the constant obtained for the pin distance 24 cm, the mean for E_{24} was 1.85×10^3 kgf/mm^2; for the pin distance of 21 cm and the constant 1/66, the mean for E_{21} was 1.75×10^3 kgf/mm^2. The corresponding values for the relative experimental error were ± 8 and ± 9 per cent for a single measurement.

VI. Clinical application

MATERIAL

It was originally intended to record the development of stability in all cases of new fractures of the tibial shaft referred to this hospital, but it was soon found that some such fractures displayed good stability after as little as 6—8 weeks of immobilization and there were therefore no therapeutic grounds for performing measurements. Still less was it justified to shorten the first period of immobilization before the fracture had become stable merely to obtain values for the investigation. On the other hand, there were patients in whom healing was slow and did not occur in the usual time, where, to judge from the radiographs, the formation of callus was unsatisfactory of where there was delayed union or risk of pseudarthrosis. It is for such cases that the method was intended as an aid in the evaluation of the healing and the progress of the treatment. The first measurement of the stability of the fractured tibia was therefore not performed until a period had elapsed after the fracture; this ranged from 6 to 39 weeks, with an average of 19. Because of the risk of infection when the pins were inserted, no examination was made in the case of infected leg wounds. When the fracture was located in the extreme distal or proximal part of the tibia, so that the pins had to be inserted quite near the respective joints, no examination could be made because the malleolar and condylar areas afford an unreliable hold.

The material for the study comprised 40 patients with shaft fractures of the tibia, 3 of them with pseudarthroses. Measurements on the tibia of the sound leg of a patient with a trimalleolar fracture were included. Examinations were repeated on two or more occasions in 29 of the patients.

The mean age of the 26 men was 38 ± 16 years (range 17—66) and of the 14 women 48 ± 15 years (range 18—75). The difference between the means is not statistically significant, but the recorded difference points to the possible existence of different external factors responsible for the fracture in men and women.

One patient had a two-level fracture, the middle fragment being about 15 cm long. In 5 patients the shaft fracture was located proximally and in the other 34, distally. Seven were open fractures. In 4 cases osteosynthesis had been performed prior to the first measurement and 5 patients underwent a total of 7 operations after this measurement.

The first measurement of the deflection of the tibia was made on average 19 weeks (6—39) after the fracture or the most recent operation. Three patients with pseudarthroses were not included because the treatment had once been regarded as completed. The mean interval between the measurements was 8 weeks (range 3—24); the average was raised considerably by the long time often left between the last two measurements.

Because, at the beginning of this investigation the formula for the moment of inertia of the tibia (eqn. [8]) had not been calculated, the radiographic examination included no films taken with projections enabling the moment of inertia of the bone to be measured. For this reason the moment was not determined for all the stability examinations. So as not to increase the risk of infection, no pin that had loosened and passed out through the skin was re-inserted in the bone.

In some cases a pin loosened when the load was less than 5 kgf, and then the value was extrapolated to 5 kgf so as to enable a comparison to be made with those obtained in subsequent measurements. As the load-deflection curves were fairly linear, extrapolation would not appreciably increase the error of measurement. The extrapolated values are marked with an asterisk in the text and curves. All the curves from the measurements performed are presented.

MEASUREMENTS ON THE INTACT TIBIA

As the two tibias of any one person displayed close similarities and had practically the same flexural rigidity (EI), it is possible from the deflection and moment of inertia of the sound tibia to obtain an impression of the flexural rigidity of the other bone before the fracture. The relation between the observed deflections of the fractured and sound tibias affords a criterion of the stability of the fracture during the course of healing. By repeating the determinations of both deflection and moment of inertia the reproducibility of the clinical measurements could be ascertained. At the same time these values indicate the suitability of the method for determining the modulus of elasticity *intra vitam*.

The reported values relate to the deflection for a pin distance of 24 cm and a load of 5 kgf. In the case of two patients in whom measurements were performed with a pin distance of 21 cm the values were converted to corresponding ones for 24 cm.

The values recorded for the sound tibia are presented in table 8. Where there were two or more values, the variation due to the error of measurement could be separated from the biological variation by means of an analysis of variance. In this case there was an hierarchic classification with case and measurement as sources of variation (*Snedecor* & *Cochran*, Chapter 10).

Results

The male and female patients were assigned to separate groups. The mean observed deflection of the sound tibia was 97 μm for the men and 149 μm for the women. The corresponding relative errors for a single measurement were ±5 and ±9 per cent.

In the statistical analysis of the results 3 values for each of cases 18, 34, and 41 were omitted (indicated in parentheses in table 8) for the following reasons. For the first measurement on case 18 the middle pin entered obliquely to the side. The middle pin was carefully loosened, but on re-insertion it followed the same track as before. This was reflected in the recorded curve (Fig. 12, curve A), which, as expected, showed a larger deflection than the subsequent ones.

For case 34 the first deflection recorded differed from the subsequent ones owing to the position of the instrument; for a check of the radiographs showed that at the first registration the instrument was 2 cm proximal of its position for the later measurements. The distance of the middle pin from the narrowest part of the tibia for the first registration was 53 mm and for the four subsequent ones 31, 31, 34 and 30 mm. The large difference may be due to the long interval of 24 weeks between the first and second examinations.

Case 41 was an early one in the investigation, when no radiographs were taken with pin in place. In the first measurement the distal pin was located too far distally on the fractured tibia and the instrument was therefore intentionally placed at a more proximal level in the following examinations.

The values of the moment of inertia of the smallest section calculated from eqn. [8], $I_{min\ appr} = 0.042\ bd^3 - 0.049\ a^4$, are presented in table 9. The model for the statistical analysis is the same as for the analysis of the deflection. The mean moment of inertia for men was 0.94 cm^4 and for women 0.58 cm^4. The corresponding relative errors for a single measurement were ±4 and ±5 per cent.

TABLE 8 *Recorded deflection for a load of 5 kgf on the intact tibia. (Cases with two or more values are used for the analysis of variance.)*

		Recorded deflection, microns									
Male cases											
1	L.H.	92	98	95							
2	H.W.	107	109	104							
3	S.S.	80	80	86	87	86					
4	L.B.	75	71	76	82						
5	K.F.	83	76	80	71	87					
6	L.L.	104	83	93	114						
7	F.S.	93	78	93							
8	N.S.	124	132	110	109						
9	G.W.	118	120	106	106						
10	G.S.	81	88	84	90						
11	G.C.	84	100	96							
12	L.P.	75	70	82	93	98	92	95	86	97	90
13	E.S.	86	96	91	90	96	87	79	89		
14	S.G.	88	95	86	94						
15	N.G.	117	130	131	131	122					
16	Å.D.	94	93	86							
17	R.J.	122	121	117	117						
18	Å.B.	(136)	107	117	124	111					
19	S.K.	68									
20	H.M.	145									
21	H.H.	137									
22	A.C.	105									
23	L.H.	85									
24	B.Ö.	54									
25	I.A.	86	93								
26	E.J.	90									
Female cases											
27	I.S.	130	125	130							
28	V.G.	135	132	141							
29	V.L.	116	111	110	113	116	132				
30	M.N.	123	124	117							
31	K.E.	158	179	189	176	183					
32	U.K.	139	143								
33	C.S.	167	175	194	168						
34	K.Å.	(133)	159	168	165	162					
35	A.L.	138									
36	K.W.	124									
37	V.N.	175	225								
38	E.R.	143	130								
39	A.B.	96									
40	B.H.	155									
41	M.Z.	(111)	147	157							

	Degrees of freedom	Mean square
Source of variation		
Case	18	951
Measurement[a]	62	23

[a]Relative error for a single measurement $100(23)^{1/2}/97 = \pm 5$ per cent

	Degrees of freedom	Mean square
Source of variation		
Case	10	2382
Measurement[a]	25	146

[a]Relative error for a single measurement $100(146)^{1/2}/149 = \pm 8$ per cent

TABLE 9 *Moment of inertia. (Cases with two or more values are used for analysis of variance).*

		Moment of inertia, cm^4							
Male cases									
1	L.H.	.817	.854	.869					
2	H.W.	.759	.801	.756					
3	S.S.	1.186	1.251	1.307	1.238	1.210			
4	L.B.	.806	.886						
5	K.F.	1.045							
6	L.L.	.886							
7	F.S.	.807							
8	N.S.	—							
9	G.W.	—							
10	G.S.	1.158	1.138	1.116					
11	G.C.	.936	.781						
12	L.P.	1.237	1.299	1.225	1.314	1.298	1.271	1.246	
13	E.S.	1.213	1.311	1.275	1.265	1.224	1.282	1.216	1.206
14	S.G.	.965	.902	.896	.971				
15	N.G.	.749	.732	.766	.740	.755			
16	Å.D.	1.110	.993	.982					
17	R.J.	.645	.630	.666	.700				
18	Å.B.	.647	.651	.642	.681	.666			
19	S.K.	1.489							
20	H.M.	.538							
21	H.H.	1.116							
22	A.C.	.978							
23	L.H.	—							
24	B.Ö.	—							
25	I.A.	—							
26	E.J.	—							
Female cases									
27	I.S.	.722	.693	.759					
28	V.G.	.797	.721	.750					
29	V.L.	.574	.632	.640					
30	M.N.	.553	.599	.538					
31	K.E.	.411	.405	.376	.407	.408			
32	U.K.	.481	.456						
33	C.S.	.514	.469	.434	.460				
34	K.Å.	.445	.447	.495	.463	.463			
35	A.L.	.652							
36	K.W.	.640							
37	V.N.	—							
38	E.R.	—							
39	A.B.	—							
40	B.H.	—							
41	M.Z.	.666	.722						

Male cases:

	Degrees of freedom	Mean square
Source of variation		
Case	12	0.2443
Determination[a]	40	0.0016

[a]Relative error for a single determination $100(0.0016)^{1/2}/0.94 = \pm 4$ per cent

Female cases:

	Degrees of freedom	Mean square
Source of variation		
Case	8	0.0577
Determination[a]	21	0.0174

[a]Relative error for a single determination $100(0.0174)^{1/2}/0.58 = \pm 5$ per cent

The modulus of elasticity of the sound tibia

From the means of the observed deflection and moment of inertia the apparent modulus of elasticity for the men and women were calculated, namely 2.65×10^3 and 2.3×10^3 kgf/mm², respectively; with the constant 1/67 the corresponding values of E are 1.9×10^3 and 1.6×10^3 kgf/mm². The determined error in the observed deflection is the same in the corrected deflection after subtraction of the systematic error of the method. When the relative error of the corrected deflection was calculated and since the relative error in the moment of inertia was known, the relative error of the modulus of elasticity was obtained from the square root of the sum of the squares of the relative errors in the measurement of the deflection and the moment of inertia; it was ± 9 and ± 12 per cent for men and women, respectively.

The modulus of elasticity of the tibia in controls

When one tibia is fractured the other one cannot be regarded as completely normal, but may be expected to display changes such as a measure of osteoporosis. It was accordingly considered of interest to perform measurements on normal tibias *intra vitam.* The apparent modulus of elasticity was determined on both tibias in 9 voluntary subjects in sound health and with no history of leg injury. Their mean age was 25 years (range 22—31). The values of the apparent modulus of elasticity ("E") and the corresponding analysis of variance are given in table 10. The observed values constitute a 2-factor experiment with the subject and the side as sources of varia-

TABLE 10 *Analysis of variance for the apparent modulus of elasticity for both tibias in male subjects 22–31 years of age.*

	Apparent modulus of elasticity, kgf/cm²								
Case	1	2	3	4	5	6	7	8	9
Right	361,277	311,277	307,927	220,065	267,354	274,494	277,031	290,208	232,465
Left	328,747	326,531	302,111	204,195	224,523	260,339	276,369	301,676	232,727

	Degrees of freedom	Mean square $\times 10^6$	F ratio	P
Source of variation				
Subject	8	3689		
Side	1	397	$F_{1;8}=2.15$	>0.05
Error and interaction[a]	8	185		

[a]Relative error for a single determination $100(185 \times 10^6)^{1/2}/277{,}000 = \pm 5$ per cent

tion (*Snedecor & Cochran*, Chapter 12). There was no significant difference between the right and left sides. The estimated relative error for a single measurement was ± 5 per cent. With the constant 1/67, E was obtained as 2.0×10^3 kgf/mm^2.

Discussion

Measurements on the tibia *intra vitam* differ in a number of respects from those on autopsy specimens. There are several factors that can have a bearing on the magnitude of the deflection.

The living tibia is exposed to the mechanical effects of the surrounding tissues, has a higher water content and is perfused with blood, which is under pressure. The effect of these factors, together with the temperature, burdens the relative errors for the single measurements of the deflection and the moment of inertia.

The deflection is influenced by the temperature, it being greater for the warmer tibia than for the autopsy specimens. In a study of the modulus of elasticity of bone in bending Sedlin (1965) found a 6 per cent lower value at 37 than at 21°C; the difference was statistically significant. This result is consistent with those reported by Smith & Walmsley (1959), who found that the elasticity modulus is inversely related to the temperature of the bone specimen.

Unlike the specimen the vital tibia has a constant water content. It has been reported that a short period of exposure of a bone specimen to a room atmosphere is enough to increase the modulus of elesticity of bone; Smith & Walsley (1959) recorded an increase of 7 per cent after one hour; however, for a period of 10 minutes Sedlin (1965) found no change.

The presence of an intact fibula has an effect on the deflection of the tibia. This was tested as follows: The deflection of the tibia on autopsy specimens was registered with the instrument. With this still in place the fibula was sawn off at the level of the middle pin. The values of the deflec-

TABLE 11 *Analysis of variance for the deflection of the tibia before and after resection of the fibula.*

	Degrees of freedom	Mean square	F ratio	P
Source of variation				
Specimen	12	3210		
Resection	1	30	$F_{1;12}=5.45$	$0.05>0.01$
Error and interaction	12	5.5		

tion after and before resection of the fibula were examined by analysis of variance. They constitued a 2-factor experiment with subject and resection as sources of variation (*Snedecor & Cochran*, Chapter 12). The analysis showed an almost significant increase in the deflection after resection (Table 11). This increase was on average 2 per cent of the deflection before resection.

If the modulus of elasticity of the sound tibia varies during the course of the investigation, there will be an increase in the error for the reproduced values of deflection. To judge from the observed deflection of the sound tibia, there was no such variation.

The effect on the deflection of varying distance from the middle pin to the narrowest part of the sound tibia was tested on each patient separately. Where there were at least two observations the slope of the regression line was determined by the method of least squares, with the deflection on the axis of y and the distance of the middle pin from the narrowest part of the tibia on the axis of x. The regression coefficient was negative in 15 cases and positive in 7 but there was no statistically significant deviation from the horizontal position of the line. The fact that in the measurements an attempt was made to obtain a constant value of x might explain why the sign test was not significant.

No explanation of the difference in the relative error in the deflection for women and men can be offered, unless it might be ascribed to the greater difficulty in obtaining a firm hold for pins in women over 50 years, possibly because of osteoporosis and a thinner cortical layer.

In the experiments on the autopsy specimens from women it also seemed to be more difficult to fix the pins in the case of the pin distance of 24 cm. This may have been due to the tendency for the tibia to be shorter in the women than the men, with respective averages of 34 and 38 cm. Thus, a pin distance of 21 cm would presumably be more suitable for measurements on the female tibia.

It is obviously more difficult to determine the midline on the medial surface of the tibia *intra vitam*, but otherwise the experimental conditions, with the mentioned exceptions, were much the same as for the autopsy specimen; the views concerning measurements on the specimens presented in the *Discussion* also apply to the tibia *intra vitam*.

The reproducibility of the approximate moment of inertia was acceptable. It would be difficult to attain a higher accuracy without making the radiologic examination much more time-consuming, but, as said before (page 15), a determination of the moment on some more levels of the tibia would give a more reliable value of the moment of inertia of the tibia.

CLINICAL INVESTIGATION

In this section the patients on whom the stability measurements were performed are presented. The relevant data constitute the basis for the evalution of the principle and the method for assessing the stability of healing fracture of the tibia.

The values for the deflection of the fractured tibia were examined as regards their correlation with time and the clinical findings and results.

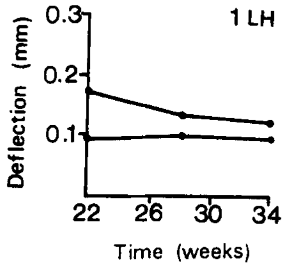
Measurements on the fractured tibia were not performed until the callus had become resilient. Only in the first case (no. 25) were registrations performed while the fracture were quite unstable. The information yielded by the recorded curves was of little value for two reasons: firstly, the clinical findings relating to the stability were reliable and, secondly, the load-deflection curve is easily influenced by the weight of the very mobile fracture fragments which loads the force transducer.

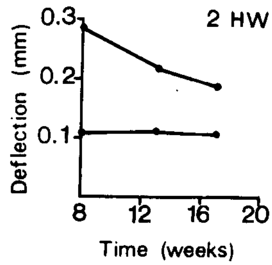
The deflections recorded for a load of 5 kgf on the fractured tibia, v_{tot}, and the intact tibia, v_t, at various intervals after fracture are presented in table 12 (page 73), together with the calculated quotient s/E_f or s/E_fI_f. The deflections of both tibias are also presented in graphs with the following case reports.

Case reports

CASE 1 L.H. Bricklayer, 20 years; upper half of the tibia fractured in a road accident. Long leg cast for 22 weeks at another hospital; during last few weeks partial weight bearing, one crutch. *Twenty-two weeks.*—Responsible surgeon doubtful whether cast might be removed; fracture nearly stable. Radiographs showed distinct callus but fracture line clearly visible. Deflection 0.172 mm, $s/E_f=0.44$. No cast, gradual weight bearing recommended. *Twenty-eight weeks.*—Had used a crutch outdoors but full weight bearing indoors. Fracture clinically stable; deflection 0.138 mm, $s/E_f=0.23$. *Thirty-four weeks.*—Full weight bearing also outdoors. Deflection 0.128 mm, $s/E_f=0.18$. Returned a month later because of occasional falls, but the fractured leg had given no trouble. Radiographs: increased amount of callus, fracture line less clear.

CASE 2 H.W. Lorry driver, 25 years; spiral fracture of tibia from falling downstairs. Long leg cast at another hospital. *Eight weeks.*—Fracture clinically unstable. Radiographs: Loose callus; large parts of the fracture line visible. Deflection 0.282 mm; $s/E_f=0.36$. Walking boot cast; weight bearing recommended. *Thirteen weeks.*—Fracture clinically stable. Radiographs: increasing callus. Deflection 0.217 mm, s/E_f= 0.19. Walking boot cast continued, increased weight bearing advised. *Seventeen weeks.*—Deflection 0.190 mm, s/E_f=0.15. Full weight bearing continued, no cast. Fracture regarded as united, fulltime work resumed a few weeks later.





CASE 3 S.S. General labourer, 53 years; proximal and distal fractures of tibia with 14 cm long middle fragment, result of road accident (Fig. 14). Long leg cast. *Seventeen weeks.*—Two crutches; slight tenderness over the distal fracture. Fracture almost stable; deflection 0.249 mm, s/E_f=0.27, calculated only for lower fracture, upper one regarded as stable. Walking boot cast, two crutches, gradual weight bearing advised. *Twenty-one weeks.*—Because of pain no increased weight bearing. Radiographs: increasing callus. Deflection 0.193 mm, s/E_f=0.18. Walking boot cast, one crutch. *Twenty-seven weeks.*—Deflection 0.118 mm, s/E_f=0.06. *Thirty-one weeks.*—Had used two crutches, but occasionally full bearing. Deflection 0.098 mm, s/E_f=0.02. Union regarded as complete.

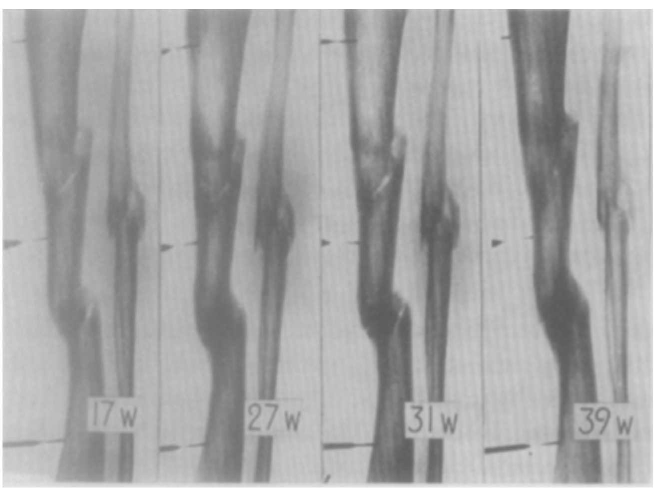
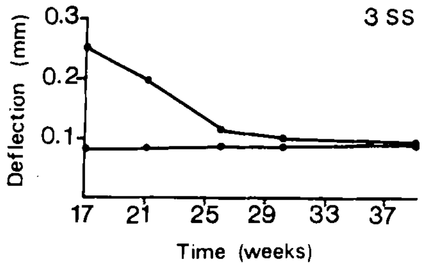
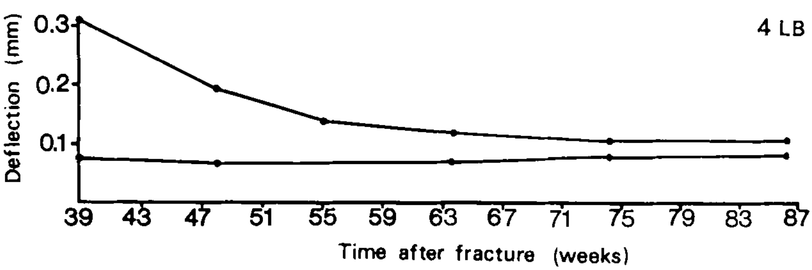


FIGURE 14 *Radiographs of fractured tibia taken 17—39 weeks after the fracture.* Case 3 S.S.

Radiographs: increased callus. Unwilling to return to strenuous work. *Thirty-nine weeks.*—Deflection 0.095 mm, $s/E_f=0.02$ mm. Referred for occupational therapy.

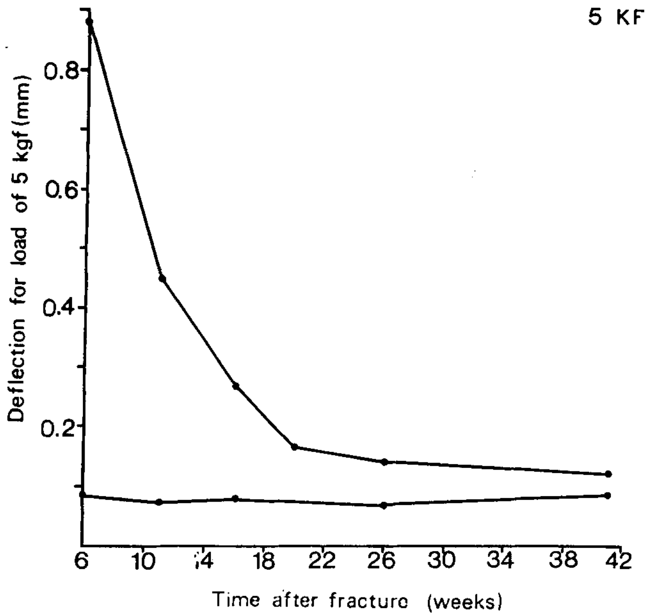


CASE 4 L.B. Stores supervisor, 58 years; spiral fracture of tibia through fall in snow. Long leg cast. *Thirty-nine weeks.*— Fracture almost stable. Radiographs: incipient callus. Deflection 0.308 mm, $s/E_f=0.29$. Long walking boot cast, gradual weight bearing advised. *Forty-eight weeks.*—Fracture loaded in the cast without trouble. Radiographs: incipient consolidation, but large parts of the fracture line still visualized. Fracture clinically almost stable. Deflection 0.192 mm, $s/E_f=0.14$. Cast not removed and increased weight bearing, one crutch. *Fifty-five weeks.*— Fracture clinically stable. Radiographic appearances unchanged. Deflection 0.140 mm, $s/E_f=0.08$. Cast removed, gradual weight bearing. *Sixty-four weeks.*—Full weight bearing, union regarded as complete. Deflection 0.119 mm, $s/E_f=0.06$. *Seventy-four weeks.*—Deflection 0.106 mm, $s/E_f=0.04$ mm. *Eighty-six weeks.*—Deflection 0.116 mm, $s/E_f=0.05$. Returned to work.



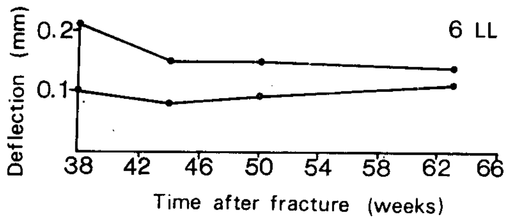
CASE 5 K.F. Clerk, 22 years; transverse fracture of upper third of tibia caused by a kick received during football. Long leg cast. *Six weeks.*—Clinically unstable. Radiographs: very loose incipient callus. Deflection 0.882 mm, $s/E_f=2.04$. *Eleven weeks.*—Fracture unstable. Radiographs: callus forming a bridge in front, but at back loose. Deflection

0.450 mm, s/E_f=0.94. Long walking boot cast, gradual weight bearing advised. *Sixteen weeks.*—Fracture almost stable. Radiographs: increased callus, but fracture line distinctly visualized. Deflection 0.278 mm, s/E_f=0.50. Two crutches without the cast; gradual weight bearing advised. *Twenty weeks.*—Fracture clinically stable. Radiographs: unchanged callus. Deflection by extrapolation 0.170* mm, s/E_f=0.22. One crutch recommended. *Twenty-six weeks.*—Patient had used one crutch outdoors and full weight bearing indoors. Radiographs: unchanged callus. Deflection 0.148 mm, s/E_f=0.17. Fracture considered to be united; returned to work 6 weeks later. *Forty-one weeks.*—Radiographs showed increased callus. No complaints. Deflection 0.129 mm, s/E_f=0.13.

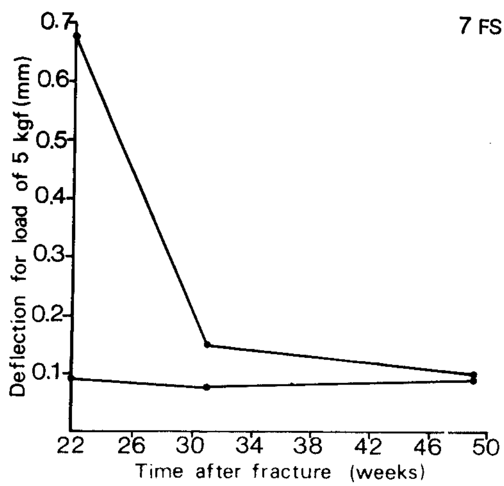


CASE 6 L.L. Stores supervisor, 43 years; short oblique fracture of tibia from road accident. Long leg cast. Repeated radiographic examinations had shown no increase in callus. Patient referred to this hospital for operation to hasten healing. *Thirty-eight weeks.*—Fracture clinically almost stable. Radiographs: callus laterally via an intermediate fragment. Deflection 0.209 mm, s/E_f=0.21. Decided to await result of further examination before deciding on operation to accelerate union. Cast removed; two crutches increasing weight bearing. *Forty-four weeks.*—Had walked up to 1 km a day with one crutch. Fracture clinically stable. Radiographs:

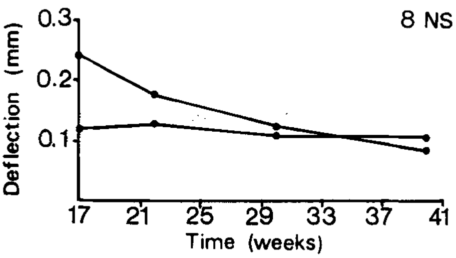
evident callus on the lateral aspect, almost none medially, large parts of the fracture line visualized. Deflection 0.150 mm, $s/E_f = 0.08$. *Fifty weeks.*—After full weight bearing, no trouble. Radiographs: unchanged callus. Deflection 0.147 mm, $s/E_f = 0.08$. Full-time work resumed after 6 weeks. *Sixty-three weeks.*—Deflection 0.143 mm, $s/E_f = 0.07$.

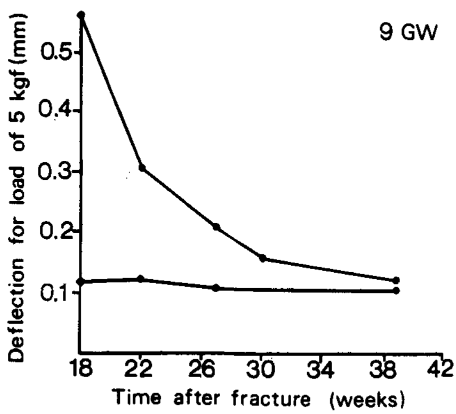


CASE 7 F.S. Student, 20 years; open transverse fracture of tibia due to an explosion. Extension treatment 4 weeks; long leg cast 12 weeks. Walking with two crutches but no weight on leg. Because of slow union, attending surgeon at another hospital proposed operation to hasten healing. *Twenty-two weeks.*—Fracture clinically unstable. Radiographs: callus bridge on lateral and posterior aspects; callus less marked on anterolateral aspect but no sclerotic change in fragment ends as in pseudarthrosis. Deflection 0.680 mm, $s/E_f = 0.79$. Walking boot cast; careful weight bearing advised. *Thirty-one weeks.*—Fracture clinically stable. Deflection 0.150 mm, $s/E_f = 0.08$. No cast, crutches withdrawn. *Forty-nine weeks.*—Full weight bearing. Radiographs: full consolidation. Deflection 0.104 mm, $s/E_f = 0.02$.



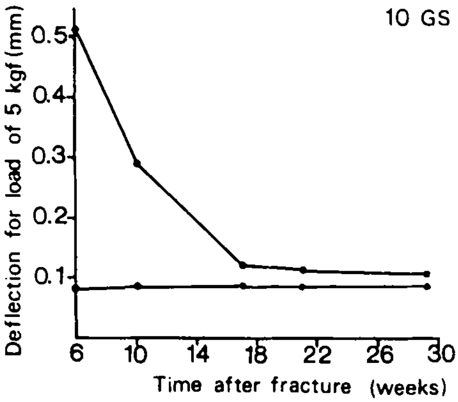
CASE 8 N.S. Student, 17 years; open distal fracture of tibia from falling tree. Fracture transfixed by two screws; long leg cast; eleven weeks later short leg cast. *Seventeen weeks.*—Fracture clinically stable. Radiographs: incipient bridging by loose callus. Deflection 0.246 mm, $s/E_fI_f=0.13$. Increased weight bearing advised. *Twenty-two weeks.*—Radiographs: increase in callus. Deflection 0.180 mm, $s/E_fI_f=0.06$. No cast, increased weight bearing; after 3 weeks, one crutch. *Thirty weeks.*—Full weight bearing. Radiographs: increasing callus. Deflection 0.130 mm, $s/E_fI_f=0.01$. *Forty weeks.*—Patient fully active. Radiographs: full consolidation. Deflection 0.99 mm, $s/E_fI_f<0$.



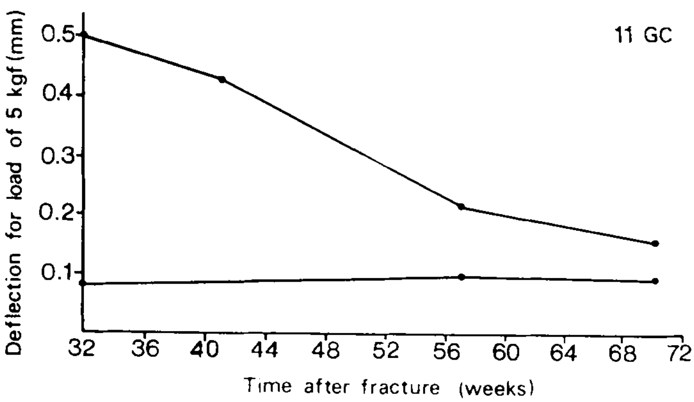


CASE 9 G.W. Student, 21 years; tibia fractured distally during vacation work. Fixation with medullary Küntscher nail, removed one month later because of proximal slipping. Long leg cast. *Eighteen weeks.*—Fracture clinically unstable. Radiographs: minimal, loose callus. Deflection 0.560 mm, $s/E_fI_f=1.10$. New cast. *Twenty-two weeks.*—fracture clinically stable. Radiographs: increased callus, but much of the fracture gap visualized. Deflection 0.305 mm, $s/E_fI_f=0.53$. No cast, gradual weight bearing; two crutches advised. *Twenty-seven weeks.*—Further stabilization. Radiographs: increased bridging callus, but fracture line still visible. Deflection 0.212 mm, $s/E_fI_f=0.24$. Increased weight bearing with only one crutch a week or two, then full weight bearing. *Thirty weeks.*—Deflection 0.160 mm, $s/E_fI_f=0.12$. *Thirty-nine weeks.*—Walks, full weight bearing, no trouble. Radiographs: increased callus, bridging on the lateral aspect. Deflection 0.125 mm, $s/E_fI_f=0.03$. Fracture regarded as united.

CASE 10 G.S. Driver, 53 years; open fracture of tibia. Long leg cast. *Six weeks.*—Clinically unstable. Radiographs: no visible callus. Deflection 0.510 mm, $s/E_f=1.13$. New cast. *Ten weeks.*—Clinically almost stable. Radiographs: incipient callus. Deflection 0.290 mm, $s/E_f=0.85$. Walking boot cast. Partial weight bearing. *Sixteen weeks.*—Fracture clinically stable. Radiographs: increased callus. Deflection 0.120 mm, $s/E_f=0.18$. *Twenty weeks.*—Radiographs: unchanged callus. Deflection 0.113 mm, $s/E_f=0.15$. No cast, gradual weight bearing, two crutches for 4 weeks, then one crutch. *Twenty-nine weeks.*—Full weight bearing. Radiographs: increasing consolidation. Deflection 0.106 mm, $s/E_f=0.11$. Full-time work resumed 2 weeks later.

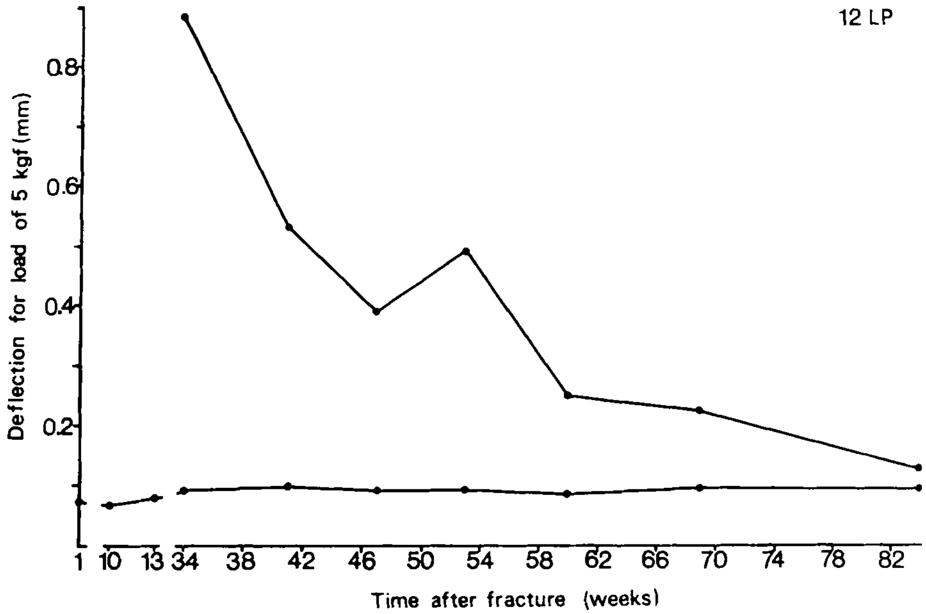


CASE 11 G.C. Engineer, 26 years; fractured tibia during skiing. Cast, delayed union. Eleven months.—Operation at another hospital, unsuccessful; twenty months new operation, sliding inlay graft; long leg cast for 7 months. Partial weight bearing with cast. Radiographs:



fracture of graft. Attending surgeon discussed further operation. *Thirty-two weeks* after latest operation.—Fracture unstable. Deflection 0.500 mm, $s/E_f=0.36$. Two crutches gradual weight bearing. *Forty-one weeks.*—For some weeks, one crutch. Radiographs: no change in callus. Fracture unstable. Deflection 0.430 mm, $s/E_f=0.29$. Continued with one crutch for a month, then full weight bearing. *Fifty-seven weeks.*—Fracture clinically stable. Radiographs: slight increase of callus. Deflection 0.218 mm, $s/E_f=0.13$. After a month work resumed. No residual symptoms. *Seventy weeks.*—Deflection 0.162 mm, $s/E_f=0.06$.

CASE 12 L.P. Carpenter, 44 years; fracture of proximal half of tibia in road accident. Extension, transfixed long leg cast. Sound tibia measured after *one week.*—Deflection 0.075 mm. *Ten weeks.*—Cast renewed. Fracture too unstable for measurement. Sound tibia, deflection 0.070 mm. *Thirteen weeks.*—Likewise; fracture unstable. Radiographs: large fracture gap. Sound tibia, deflection 0.084 mm. Operation to hasten union considered, but patient preferred to continue with cast. Eight months after fracture, instability persisted but careful loading of leg in a long leg cast permitted. *Thirty-four weeks.*—Fracture unstable. Radiographs: slight increase of callus from negligible (Fig. 15). Deflection 0.880 mm, $s/E_f=1.75$. Long walking boot cast, gradual weight bearing continued. *Forty-one weeks.*—Clinically unstable. Radiographs: no change. Deflection 0.530 mm, $s/E_f=0.96$. New cast. *Forty-seven weeks.*—Clinically unstable. Radiographs unchanged. Deflection 0.392 mm, $s/E_f=0.67$. Because the middle pin loosened, radiographs were taken without pins in place. Considerable stiffness in the knee and ankle owing to the long period of immobilization; cast removed. Two crutches; instructed to put weight on leg carefully; despite this warning, had sometimes put full weight on the leg though without pain or aching. *Fifty-three weeks.*—Fracture unstable. Radiographs: unchanged callus. Deflection 0.493 mm, $s/E_f=0.88$. Because of this increase and to prevent development of pseudarthrosis, use of knee cast. Partial weight bearing permitted, but with two crutches. *Sixty weeks.*—Fracture unstable. Deflection 0.250 mm, $s/E_f=0.35$. Knee cast continued, increased weight bearing. *Sixty-nine weeks.*—Occasional full weight bearing, no pain. Fracture clinically stable. Radiographs: increase in callus. Deflection 0.226 mm, $s/E_f=0.30$. Cast removed, one crutch withdrawn two weeks before last measurement. *Eighty-four weeks.*—Knee mobility 180—80°, ankle 75—135°. Circumference of the thigh 1 cm less on side of fracture. Deflection 0.127 mm, $s/E_f=0.07$. One month later full-time work resumed. Union complete; no residual disability.



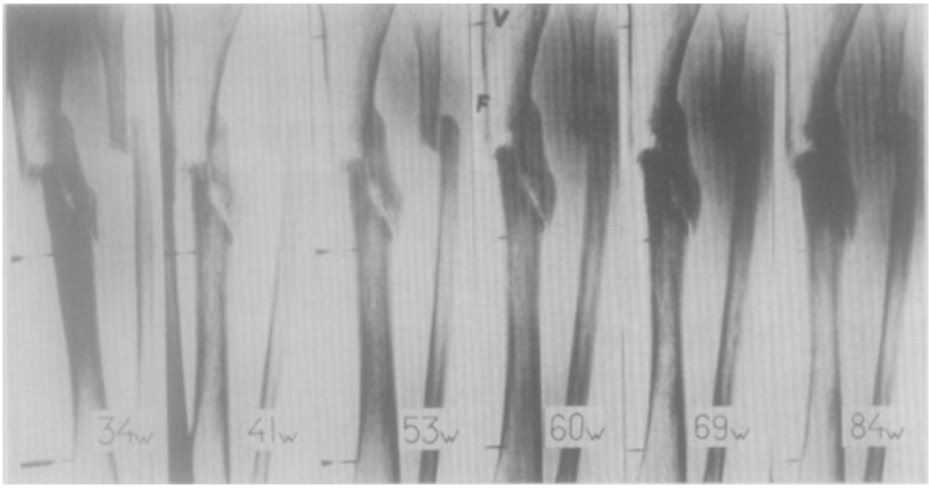
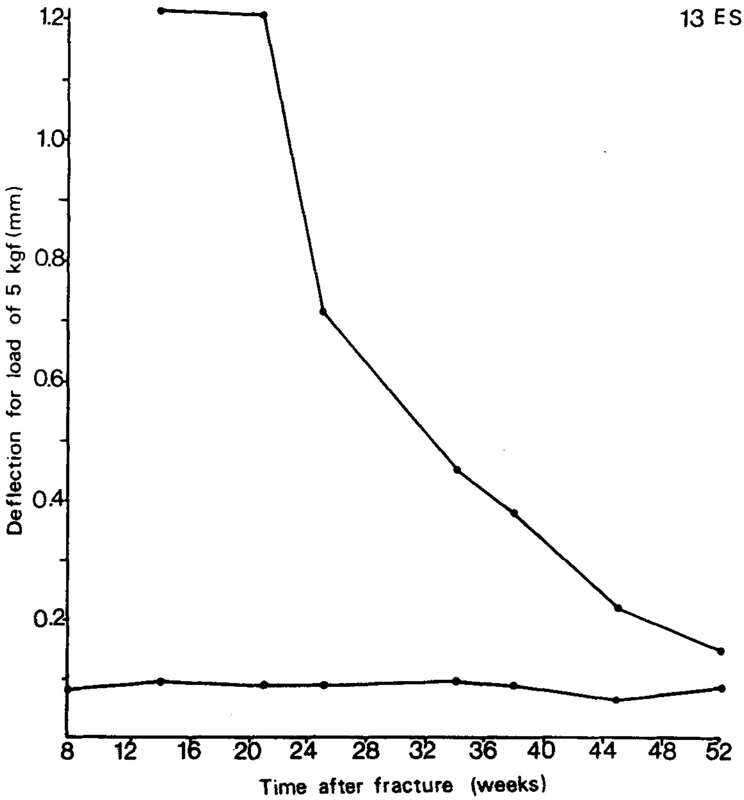


FIGURE 15 *Radiographs of fractured tibia taken 34—84 weeks after fracture.* Case 12 L.P.

CASE 13 E.S. Smith, 66 years; retired on sick pension. Involved in road accident. Treated at another hospital for skull fracture, concussion, delirium tremens and fracture of tibia. Long leg cast. *Eleven weeks.*—When cast changed, fracture too unstable to be measured. Sound



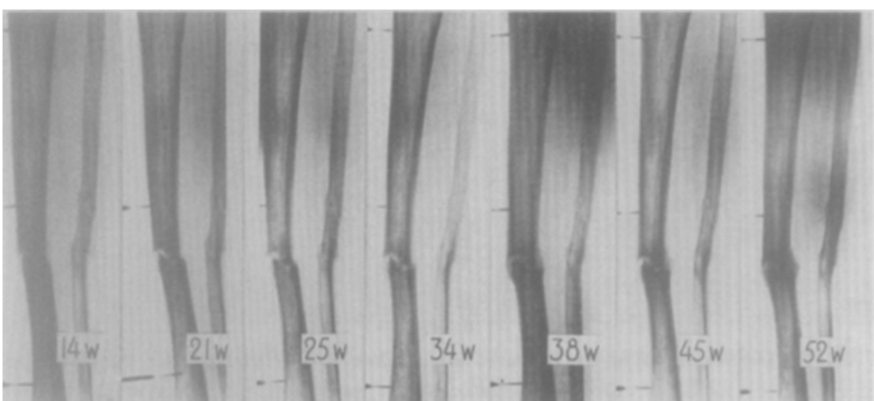


FIGURE 16 *Radiographs of fractured tibia taken 14—52 weeks after fracture.* Case 13 E.S.

tibia, deflection 0.086 mm. *Fourteen weeks.*—Fracture still quite unstable. Radiographs: no evidence of consolidation (Fig. 16); corresponding original load-deflection curves representing course of healing in figure 17. Deflection 1.210 mm, $s/E_f = 1.55$. *Twenty-one weeks.*—No change; deflection 1.200 mm, $s/E_f = 1.57$. *Twenty-five weeks.*—Fracture unstable. Radio-

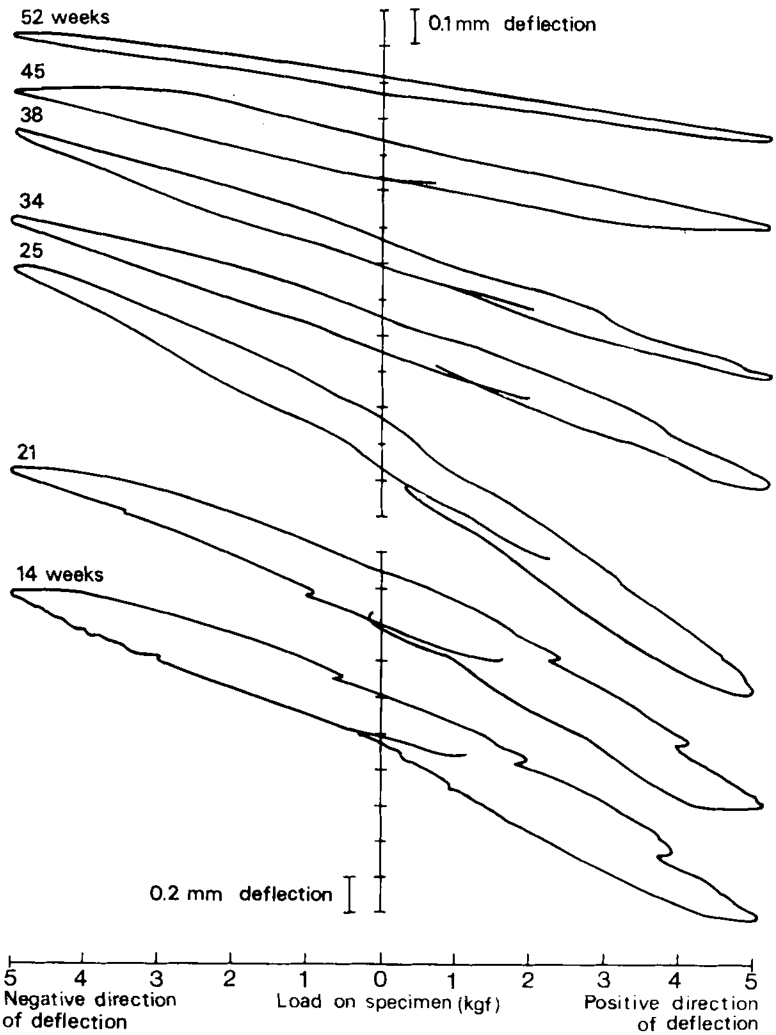
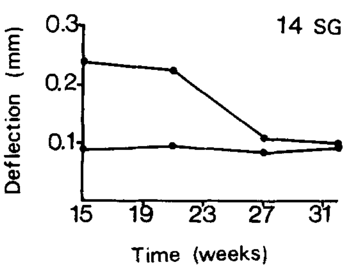


FIGURE 17 *Load–deflection curves for fractured tibia taken 14—52 weeks after fracture.* Case 13 E.S.

Note that in the curves 14 and 21 weeks after fracture the scale for deflection is different from that for the other curves.

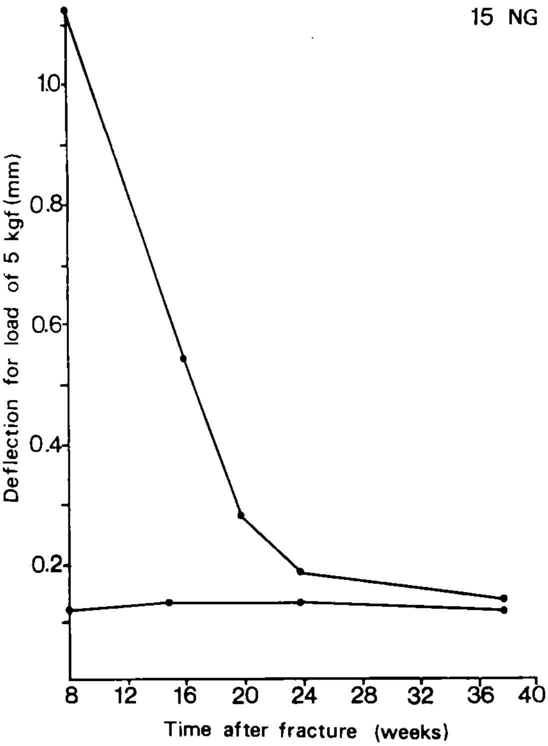
graphs unchanged. Deflection 0.718 mm, $s/E_f=0.90$. Long walking boot cast. Gradual weight bearing. *Thirty-four weeks.*—Fracture unstable. Radiographs: slight callus on anterior aspect. Deflection 0.446 mm, $s/E_f=0.49$. Patellar-tendon-bearing (PTB) cast; two crutches and gradual weight bearing. *Thirty-eight weeks.*—Clinically almost stable. Deflection 0.376 mm, $s/E_f=0.41$. Continued PTB cast, increased weight bearing. *Forty-five weeks.*—One crutch, occasionally full weight bearing; fracture stable. Radiographs: increase in callus. Deflection 0.222 mm, $s/E_f=0.19$. Increased weight bearing without cast advised. *Fifty-two weeks.*—A year after accident, full weight bearing, slight limp because of pain in the ankle. Knee mobility 180—90° ankle 85—120°. Deflection 0.149 mm, $s/E_f=0.08$. Fracture considered united. Radiographs showed consolidation.

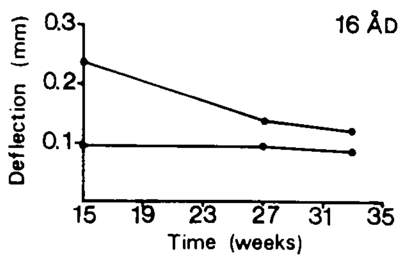
CASE 14 S.G. Building worker, 47 years; fractured tibia in a fall in street. Transfixion and long leg cast. *Fifteen weeks.*—Fracture unstable. Radiographs: no callus. Deflection 0.243 mm, $s/E_f=0.41$. PTB cast, partial weight bearing. *Twenty-one weeks.*—Fracture almost stable, clinically. Radiographs: no definite bridging by callus; fracture line clearly visible. Deflection 0.222 mm, $s/E_f=0.26$. Tenderness around fracture. Cast retained. *Twenty-seven weeks.*—Clinically stable. Radiographs: fracture line still evident, but callus increased. Deflection 0.110 mm, $s/E_f=0.06$. No cast; gradual weight bearing, at first two crutches. *Thirty-two weeks.*—Fracture regarded as clinically healed. Radiographs: increase of callus, but much of fracture line visualized. Deflection 0.100 mm, $s/E_f=0.02$.



CASE 15 N.G. Draughtsman, 20 years; fractured tibia during skiing. Long leg cast. *Eight weeks.*—Clinically unstable. Radiographs: loose callus. Deflection 1.120 mm, $s/E_f=1.92$. PTB cast; two crutches. *Fifteen weeks.*—Fracture unstable. Radiographs: increase of callus. Deflection 0.540 mm, $s/E_f=0.73$. *Twenty weeks.*—Weight bearing gradu-

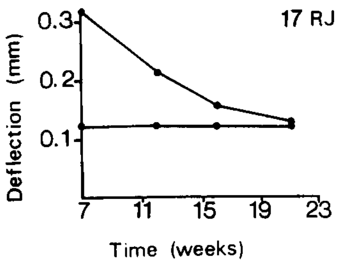
ally increased, occasional full weight bearing for a few steps. Fracture almost stable clinically. Deflection 0.278 mm, s/E_f=0.25. Exercise mobility; first with two crutches, later one, without cast. *Twenty-four weeks.*—Last seven days, full weight bearing. Walks of up to 90 min. Fracture regarded as healed. Deflection 0.186 mm, s/E_f=0.10. *Thirty-eight weeks.*—No residual symtoms. Deflection 0.139 mm, s/E_f=0.02.

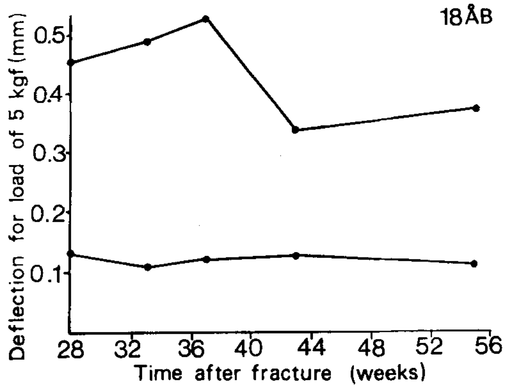




CASE 16 Å.D. Typographer, 35 years; fractured tibia in fall. Long leg cast. *Fifteen weeks.*—Clinically unstable. Radiographs: loose callus and clearly visualized fracture gap. Deflection 0.234 mm, s/E_f=0.63. Same day, open reduction and transfixion of fracture with AO screws and long leg cast. *Twenty-seven weeks.*—Fracture clinically stable. Radiographs: good callus. Deflection 0.136 mm, s/E_f=0.18. Cast removed; gradual weight bearing. *Thirty-three weeks.*—Full weight bearing; no residual trouble. Radiographs: consolidation. Deflection 0.119 mm, s/E_f=0.11. Fracture regarded as healed.

CASE 17 R.J. Porter for block of flats, 53 years; fractured skull, ribs and tibia in a road accident. Long leg cast. *Seven weeks.*—Fracture clinically unstable. Radiographs: loose callus. Deflection 0.317 mm, $s/E_f = 0.49$. PTB cast, partial weight bearing. *Twelve weeks.*—Fracture clinically stable. Radiographs: slow increase of callus, minimal on posterior and lateral aspects. Deflection 0.212 mm, $s/E_f = 0.22$. *Sixteen weeks.*—Fracture clinically stable. Radiographs: unchanged callus. Deflection 0.154 mm, $s/E_f = 0.09$. No cast. *Twenty-one weeks.*—Full weight bearing. Radiographs: good callus. Fracture line less distinct. Deflection 0.126 mm, $s/E_f = 0.02$. Union of fracture complete.





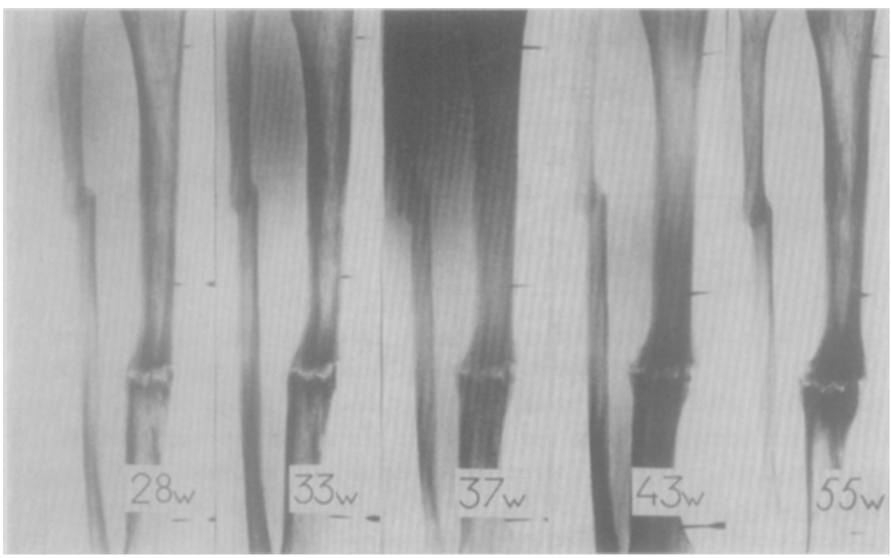


FIGURE 18 *Radiographs of fractured tibia taken 28—55 weeks after fracture.* Case 18 Å.B.

CASE 18 Å.B. Dock worker, 42 years; fractured tibia in road accident. Long leg cast. Fracture malunited. Varus position, angle more than 20°. Six months later, walking without support, persistent symptoms owing to foot deformity. Corrective osteotomy attempted; long leg cast worn. *Twenty-eight weeks* after operation.— Osteotomy unstable. Radiographs: poor callus (Fig. 18). Deflection 0.452 mm, s/E_f=0.30. PTB cast; gradual weight bearing. *Thirty-three weeks.*—Clinically unstable. Radiographs: unchanged status. Deflection 0.487 mm, s/E_f=0.35. Continued PTB cast. *Thirty-seven weeks.*—Clinically unstable. Full weight bearing in cast. Deflection 0.522 mm, s/E_f=0.34. PTB cast continued. *Forty-three weeks.*—Some clinical increase in stability—almost stable. Radiographs: fracture line less distinct. Deflection 0.339 mm, s/E_f=0.17. No cast. Full weight bearing. At this fourth measurement reduction in deflection first registered. *Fifty-five weeks.*—Full weight bearing. Fracture unstable. Radiographs: no increase of callus. Deflection 0.370 mm, s/E_f= 0.23. Regarded as a case of delayed healing.

CASE 19 S.K. Office worker, 39 years, amateur footballer. Transverse fracture in the middle of tibia from kick. Long leg cast. Fracture healed well and football training resumed a year later, but a new fracture or refracture occurred. New cast. *Twenty-three weeks.*—Attending surgeon doubtful whether to permit full weight bearing. Patient had been walking with crutches for two months without cast. Deflection 0.092 mm, s/E_f=0.03. Radiographs: bridging good callus. Full weight bearing advised; union regarded as complete.

CASE 20 H.M. Saw-mill worker, 21 years; suspected abuse of narcotics; on jumping from third floor window suffered vertebral compression fracture of L_3, acetabular and calcanear fractures and open tibia fracture. This did not heal normally, but valgus position gradually developed, with poor callus. Osteotomy, reduction and immobilization in cast. *Seventeen weeks.*—Fracture clinically unstable. Radiographs: poor callus. Deflection 0.583 mm, s/E_f=0.40. Walking boot cast. *Twenty-seven weeks.*—Clinically stable. Full weight bearing, walking without trouble. Unwilling to undergo further examinations.

CASE 21 H.H. Retired head waiter, 63 years; proximal fracture of tibia resulting from fall. Long leg cast. *Fifteen weeks.*—Clinically almost stable. Radiographs: bridging callus, fracture line still visible. Deflection 0.235* mm, s/E_f=0.29; extrapolation from 4 kgf. No cast. Recovery uneventful.

CASE 22 A.C. Shop-keeper, 62 years; proximal fracture of tibia through fall. Long leg cast. *Fourteen weeks.*—Fracture clinically stable. Radiographs: evident callus, but parts of the fracture line clearly visible. Deflection 0.154 mm, $s/E_f=0.18$. No cast; gradual weight bearing. *Nineteen weeks.*—Walking without support. Patient considered fracture as healed and refused further examinations. Fracture regarded as united. Returned to half-time work and 6 weeks later to full time. No residual trouble.

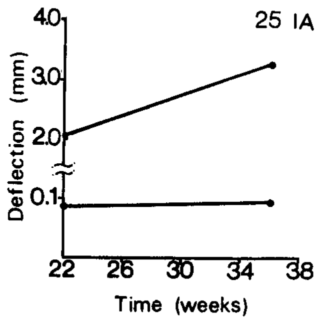
CASE 23 L.H. No regular occupation, 28 years, alcoholic; open fracture of tibia in road accident. Below-knee cast and transfixion with Kirschner wires. Skin necrosis and infection. *Ten weeks.*—Sequestroectomy performed. *Fourteen weeks.*—Fracture clinically unstable. Radiographs: poor callus. Deflection 0.221 mm. Long leg walking boot cast. Patient had no trouble with the leg and himself broke away the cast, returned 10 months later because of recurrence of osteitis in fracture. Walked without difficulty since he had removed the cast. Radiographs: united fracture.

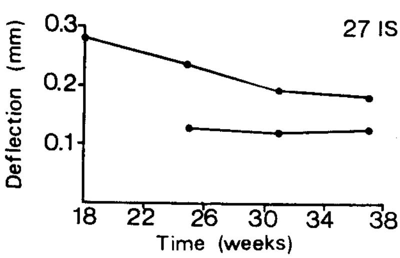
CASE 24 B.O. Seaman, 23 years; tibia fractured in accident on board ship. Open reduction and transfixion with an Egger's plate. Returned to work after 6 months. Worked for 3 years, but during military sevice leg pained on strenuous marches. At site of the four-years-old fracture radiographs disclosed thick callus about twice diameter of tibia shaft. Clinically full stability. Deflection 0.088 mm; sound tibia, 0.054 mm. Attending surgeon preferred operation. Healing normal. Returned to work 4 months later. Since fracture was stable before operation, and especially as the metal plate was left in place, there was no reason to perform further measurements of stability.

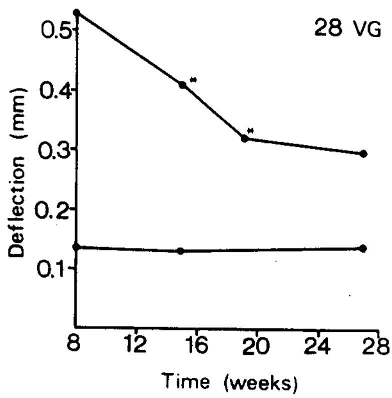
CASE 25 I.A. Shop assistent, 38 years; alcoholic; tibia fractured in road accident. Cast; no healing. *Twenty-two weeks.*—Fracture quite unstable. Deflection 2.03 mm; sound tibia 0.093 mm. Osteosynthesis same day with inlay graft and Hoffman fixation. No union. *Thirty-six weeks.*—Fracture quite unstable. Deflection 3.29 mm; sound tibia, 0.086 mm. A week later re-operation, osteosynthesis with screws and on-lay graft. A month later transferred to the hospital of home-town.

CASE 26 E.J. Electrician, 46 years; oblique fracture of tibia in fall. Reduction and long leg cast. *Twenty-two weeks.*—Fracture clinically unstable according to attending surgeon. Radiographs: evident callus. Deflection 0.119 mm, $s/E_f I_f = 0.05$. Stability measurements pointed to reliable stability, cast discontinued. Gradual weight bearing permitted. *Twenty-seven weeks.*—Fracture clinically stable; no further measurements; fracture regarded as healed. Returned to full-time work in 3 weeks.

CASE 27 I.S. Cashier, 40 years; fracture of tibia. Plaster cast. *Eighteen weeks.*—Clinically almost stable. At the first attempt to measure stability no sure hold for the distal pins obtained. Radiographs: no evidence of callus. Two days later, with different pin positions, deflection 0.280 mm, $s/E_f = 0.28$. Walking boot cast; gradual weight bearing on cast recommended. *Twenty-five weeks.*—Clinically almost stable. Radiographs: incipient formation of callus. Deflection 0.238 mm, $s/E_f = 0.22$. Walking boot cast. *Thirty-one weeks.*—Fracture clinically stable. Radiographs: incipient callus. Deflection 0.192 mm, $s/E_f = 0.13$. Full weight bearing without support. *Thirty-seven weeks.*—Fracture stable. Radiographs: increase of callus. Deflection 0.181 mm, $s/E_f = 0.09$. Fracture regarded as clinically healed, return to work recommended; five weeks later working full time.



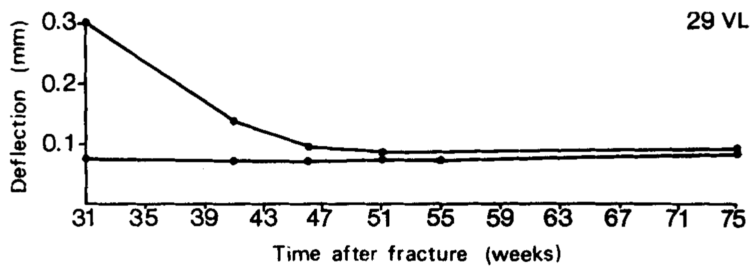




CASE 28 V.G. Clerk, 59 years, fracture of tibia. Long leg cast. *Eight weeks.*—Fracture unstable. Radiographs: incipient callus. Deflection 0.524 mm, $s/E_f = 0.86$. Long leg cast renewed. *Fifteen weeks.*—Fracture almost clinically stable. Deflection 0.410* mm, $s/E_f = 0.58$; extra-

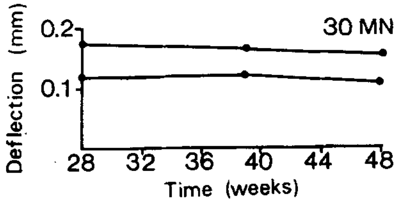
polation from 3 kgf. Walking boot cast. *Nineteen weeks.*—Fracture unstable. Radiographs: consolidation of the fracture laterally, incipient consolidation medially. Deflection 0.322* mm, $s/E_f = 0.40$, extrapolation from 3.5 kgf. Cast removed, gradual weight bearing, directed to use two crutches. *Twenty-seven weeks.*—Fracture had then been clinically stable for past month and for two weeks only one crutch had been used. Fracture regarded as healed; no support required. Deflection 0.300 mm, $s/E_f = 0.33$. Nine weeks later half-time work resumed; no residual deformity. Radiographs showed complete consolidation of shaft fracture.

CASE 29 V.L. Housewife, 36 years; transverse fracture of tibia in road accident. Long leg cast. *Thirty-one weeks.*—Clinically almost stable. Radiographs: no over-riding callus; fracture line visible. Deflection (21-cm pin distance) 0.303 mm, $s/E_f = 0.31$. Long walking boot cast. *Thirty-six weeks.*—Fracture clinically stable stability measurement unsuccessful—no radiographs to verify distance from middle pin to centre of fracture. *Forty-one weeks.*—Deflection 0.142 mm, $s/E_f = 0.10$. Cast removed. *Forty-six weeks.*—Fracture stable; full weight bearing. Deflection 0.097 mm, $s/E_f = 0.03$. *Fifty-one weeks.*—Full weight bearing, clinical union. Radiographs: still no over-riding callus on medial aspect of shaft fracture. Deflection 0.088 mm, $s/E_f = 0.02$. Recommended resume work half-time. *Seventy-five weeks.*—Recovery complete, working full-time; no disability. Radiographs: complete consolidation. Deflection 0.099 mm, $s/E_f = 0.02$.



CASE 30 M.N. Housewife, 49 years, open fracture of tibia in road accident. *Twenty-eight weeks.*—Fracture stable for last couple of months. Radiographs: poor callus development. Deflection 0.177 mm, $s/E_f = 0.04$; indicative of good stability; cast removed, weight bearing recommended. *Thirty-nine weeks.*—Because mobility reduced in both knee and ankles, patient had still not dared to put weight on the leg without support. Varus position of ankle; no crutches. Deflection 0.166 mm,

$s/E_f=0.03$. *Forty-eight weeks.*—Deflection 0.153 mm, $s/E_f=0.01$. Full weight bearing and walks advised. Three months later patient had loaded the leg fully without trouble and returned to full-time work. Radiographs: consolidation.



CASE 31 K.E. Clerk, 55 years; tibia fractured in road accident. *Thirteen weeks.*—Fracture unstable. Radiographs: incipient callus. Deflection 1.290 mm, $s/E_f=0.54$. Renewed long leg cast. *Twenty-one weeks.*—Fracture almost stable. Radiographs: increase of callus (Fig. 19). Deflection 0.760 mm, $s/E_f=0.29$; two crutches, careful loading. *Twenty-seven weeks.*—Clinically stable. Deflection 0.720* mm, $s/E_f=0.25$, extrapolation from 4 kgf; one crutch. *Thirty-three weeks.*—Deflection 0.540* mm, $s/E_f=0.17$; extrapolation from 3.5 kgf. Full weight bearing without support. *Forty weeks.*—Walked without trouble. Fracture regarded as clinically healed. Radiographs: fracture consolidated. Deflection 0.389 mm, $s/E_f=0.11$.

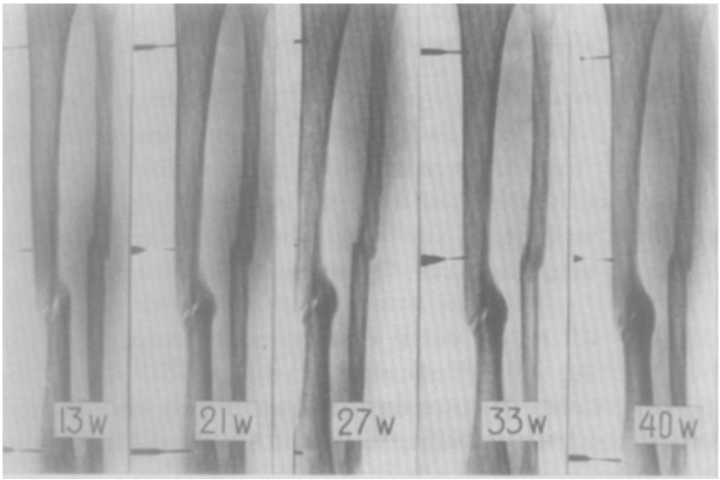
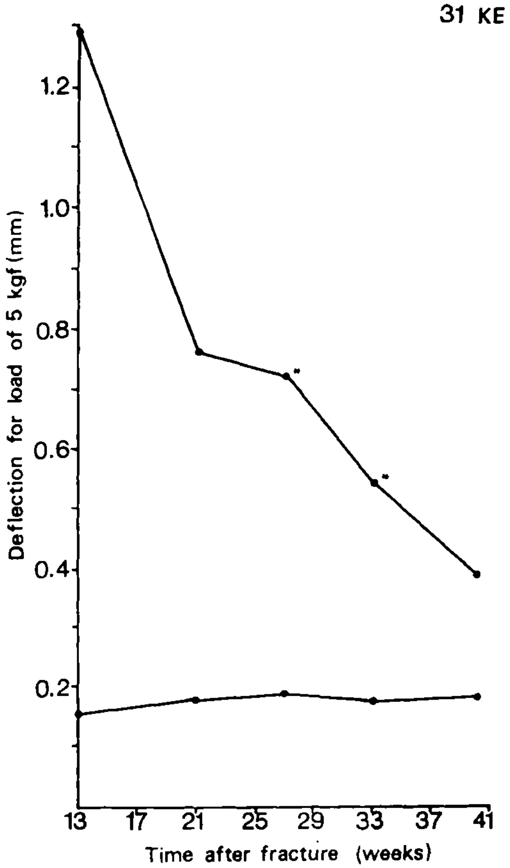
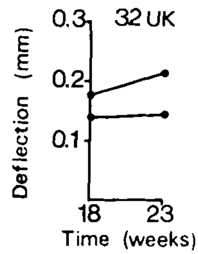


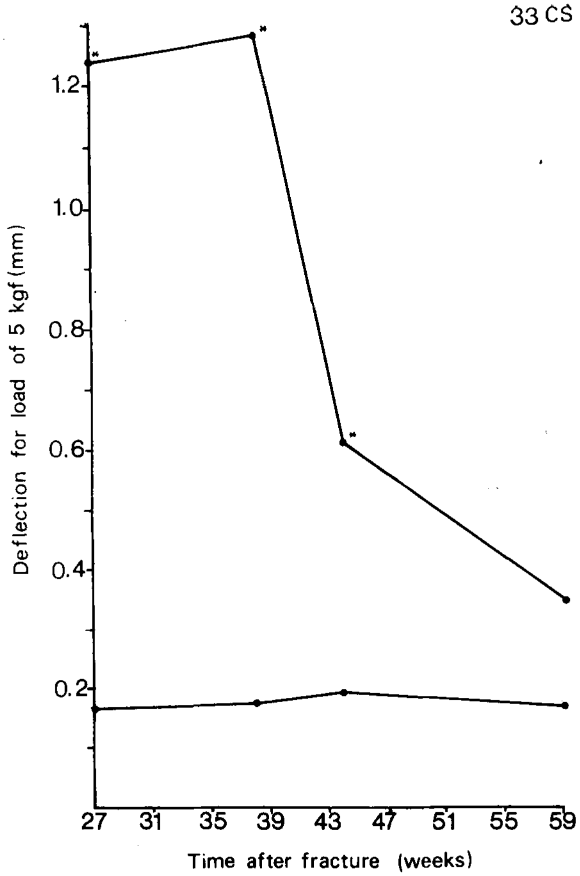
FIGURE 19 *Radiographs of fractured tibia taken 13—40 weeks after fracture.* Case 31 K.E.





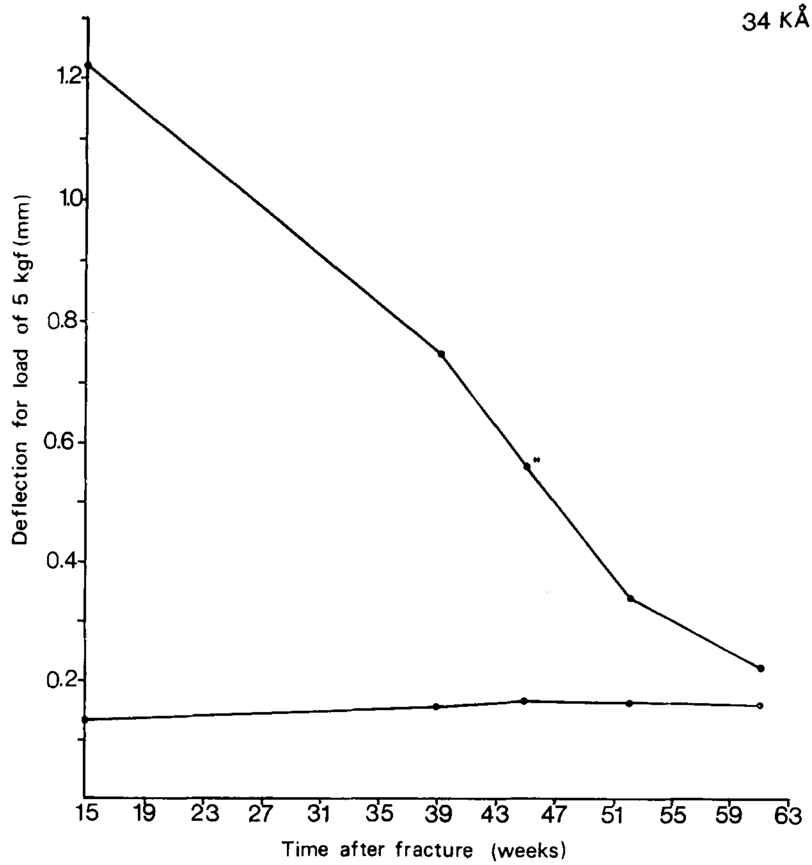
CASE 32 U.K. Dentist, 27 years, tibia fracture during skiing. Long leg cast. *Eighteen weeks.*—Fracture clinically stable. Radiographs: incomplete healing. Deflection 0.154 mm, s/E_f=0.03, indicative of stability. Cast discontinued, full weight bearing. *Twenty-three weeks.*—Deflection 0.163 mm, s/E_f=0.04. Fracture was regarded as healed.

CASE 33 C.S. Clerk, 65 years; tibia fractured in road accident. Long leg cast. *Twenty-seven weeks.*—No union, increasing valgus position. Corrective osteotomy and autologous bone graft. Prior to the operation, deflection 1.240* mm, s/E_f=0.67, extrapolation from 4 kgf. *Thirty-eight weeks.*—Eleven weeks after the operation, fracture still unstable. Deflection 1.280* mm, s/E_f=0.56, also extrapolated from 4 kgf. Long leg cast; careful loading recommended. *Forty-four weeks.*—Fracture



almost stable. Deflection 0.610* mm,s/E_f=0.20, extrapolation from 4 kgf, cast discontinued; two crutches. Sex weeks later fracture clinically stable. *Fifty-nine weeks.*—Deflection 0.349 mm, s/E_f=0.11. Walking without trouble; recovery complete. Radiographs: increasing callus during course of treatment.

CASE 34 K.Å. Housekeeper, 50 years; open tibia fracture. Long leg cast. *Fifteen weeks.*—Fracture unstable. Radiographs: no callus. Deflection 1.220 mm, s/E_f=0.86. *Twenty-two weeks.*—Open reduction. Transfixion with AO screw. Long leg cast. *Thirty-nine weeks.*—Fracture unstable. Radiographs: poor callus. Fracture line clearly visible. Deflection 0.745 mm, s/E_f=0.39. *Forty-five weeks.*—Clinically unstable. Deflection 0.560* mm, s/E_f=0.28, extrapolation from 4 kgf. PTB cast; gradual weight bearing. *Fifty-two weeks.*—Clinically stable. Radiographs:

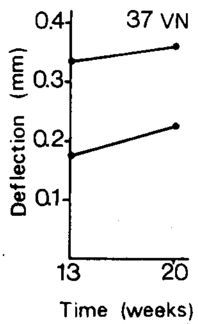


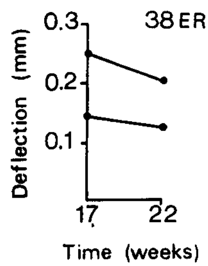
good callus. Deflection 0.340 mm, $s/E_f = 0.13$. *Sixty-one weeks.*—Walking without support; clinically healed. Radiographs: increasing callus, fracture consolidated. Deflection 0.227 mm, $s/E_f = 0.05$.

CASE 35 A.L. Office cleaner, 60 years; open tibia fracture. Open reduction, osteosynthesis with loops of wire. Long leg cast for three months; fracture regarded as healed. *Fifty-two weeks.*—Complained of pain in leg during walks. Fracture clinically stable. Radiographs: pseudarthrosis with marked sclerotic periosteal reaction. Deflection (21-cm pin distance) 0.108 mm; sound tibia, 0.092 mm; indicative of complete stability. During removal of the wires part of protecting callus chipped away, resulting in mobility between sclerotic bone ends, a pseudarthrosis. Autologous bone graft was inserted. Necrosis of wound margins, secretion, fistula. *Seventy-two weeks.*—Fistula still present. Infection in area of operation precluded further stability measurement. Radiographs: consolidation of pseudarthrosis.

CASE 36 K.W. Housewife, 44 years, tibia fractured during skiing. Long leg cast. *Twenty-nine weeks.*—Fracture clinically stable. Radiographs: incipient consolidation. Deflection 0.265* mm, $s/E_f = 0.10$. PTB cast 3 more weeks, full weight bearing without support. *Forty-nine weeks.*—Radiographs: consolidation.

CASE 37 V.N. 75 years; fractured femur (intertrochanteric) and tibia on same side in road accident. Long leg cast. *Thirteen weeks.*—Radiographs: bridging callus, mostly on lateral aspect. Cast discontinued. Deflection 0.360* mm, extrapolation from 3 kgf; sound tibia, 0.200 mm. *Twenty weeks.*—Deflection 0.335 mm. Long walking boot cast because of need to load right leg owing to intertrochanteric fracture. Removed after one week; crutches; gradual weight bearing. Both measurements pointed to good stability. Further course uneventful. *Thirty-six weeks.*—Recovered; full weight bearing.





CASE 38 E.R. Clerk, 18 years; tibia fractured in road accident. Long leg cast. *Seventeen weeks.*—Radiographs: little increase in previously minimal callus. Attending surgeon found stability doubtful. Deflection 0.252 mm, $s/E_f = 0.15$; sound tibia, 0.137 mm; indicative of stability, cast discontinued; two crutches. *Twenty-two weeks.* —Increased weight bearing. Deflection 0.210 mm, $s/E_f = 0.11$. *Twenty-eight weeks.*—Fracture clinically healed; full weight bearing.

CASE 39 A.B. Housewife, 48 years; tibia fractured in road accident. Treated at another hospital; two year later complained of pain on effort. Radiographs: pseudarthrosis; clinically unstable. Deflection 0.553 mm; sound tibia, 0.096 mm. Pseudarthrosis operation, osteosynthesis with screws and autologous bone graft. Postoperative course uneventful; normal recovery time.

CASE 40 B.H. Nurse, 50 years; tibia fractured in road accident. Reduction, long leg cast. *Thirteen weeks.*—Radiographs: minimal callus. *Sixteen weeks.*—Clinically unstable. Deflection 0.400* mm, $s/E_f I_f = 0.55$; sound tibia, 0.155 mm. Walking boot cast. Returned to former hospital for further treatment. Recovery uneventful.

CASE 41 M.Z. Clerk, 67 years; trimalleolar fracture. Patient not included in series but values included in analysis of error of determination of deflection of sound tibia and moment of inertia from formula for I_{appr} because two measurements had been performed on sound tibia and radiographs had been taken in suitable projections. Determinations performed 4 and 6 months after fracture.

Results

The values of s/E_f or $s/E_f I_f$ obtained at the examinations where the clinical evaluation of the stability of the fracture was followed by a new step in treatment or where the fracture was regarded as stable are given in the table 13. Treatment with walking boot cast was given in the interval $s/E_f = 0.66$—0.21 ($n = 17$); removal of the cast and use of two crutches was begun at the value of 0.31 ($n = 12$); the fracture was judged to be stable when $s/E_f = 0.30$—0.17 ($n = 22$) and full weight bearing was introduced at $s/E_f = 0.17$—0.10 ($n = 17$).

The healing time for the fracture varied individually and the average time after fracture until full weight bearing was resumed was 33 weeks (range 16—69) for 34 patients.

The stability of the callus is expressed by the ratio s/E_f, whose values at various intervals after fracture have been entered in figure 20. The slope of the line between two measurements gives the average rate of healing. The advantage of expressing the deflection of the callus by the ratio s/E_f instead of δ_f is that the former relates to a specific deflection per unit of inverse moment of inertia and per unit load for any particular case; the measurement δ_f is dependent on the factors governing the magnitude of the deflection—for example, the distance between the nearer outer pin and the fracture, the moment of inertia and the load.

In cases 8 and 9 the ratio $s/E_f I_f$ is plotted in figure 20. To judge from their physical constitution the value of I_f should lie on the positive side close to the mean of I_{appr} for the male cases (0.94 cm^4). This would give a value about 1.0 cm^4 for I_f.

TABLE 12 *Recorded deflection of the fractured (v_{tot}) and sound tibia (v_t) during the course of healing, and the corresponding values of s/E_f. (Italic numbers are extrapolated values.)*

Case	Weeks	v_{tot} (mm)	v_t (mm)	s/E_f (s/E_fI_f)
1 L.H.	22	.172	.092	.44
	28	.138	.098	.23
	34	.128	.095	.18
2 H.W.	8	.282	.107	.36
	13	.217	.109	.19
	17	.190	.104	.15
3 S.S.	17	.249	.080	.27
	21	.193	.080	.18
	27	.118	.086	.06
	31	.098	.087	.02
	39	.095	.086	.02
4 L.B.	39	.308	.076	.29
	48	.192	.071	.14
	55	.140	.076	.08
	64	.119	.076	.06
	74	.106	.082	.04
	86	.116	.082	.05
5 K.F.	6	.882	.083	2.04
	11	.450	.076	.94
	16	.278	.080	.50
	20	.170	.071	.22
	26	.148	.080	.17
	41	.129	.087	.13
6 L.L.	38	.209	.104	.21
	44	.150	.083	.08
	50	.147	.093	.08
	63	.143	.114	.07
7 F.S.	22	.680	.093	.79
	31	.150	.078	.08
	49	.104	.093	.02
8 N.S.	17	.246	.124	(.13)
	22	.180	.132	(.06)
	30	.130	.110	(.01)
	40	.099	.109	(<0)
9 G.W.	18	.560	.118	(1.10)
	22	.305	.120	(.53)
	27	.212	.106	(.24)
	30	.160	—	(.12)
	39	.125	.106	(.03)
10 G.S.	6	.510	.081	1.13
	10	.290	.086	.85
	16	.120	.088	.18
	20	.113	.084	.15
	29	.106	.090	.11
11 G.C.	32	.500	.084	.36
	41	.430	.091	.29
	57	.218	.100	.13
	70	.162	.096	.06
12 L.P.	34	.880	.093	1.75
	41	.530	.098	.96
	47	.392	.092	.67
	53	.493	.095	.88
	60	.250	.086	.35
	69	.226	.097	.30
	84	.127	.090	.07
13 E.S.	14	1.210	.096	1.55
	21	1.200	.091	1.57
	25	.718	.090	.90
	34	.446	.096	.49
	38	.376	.087	.41
	45	.222	.079	.19
	52	.149	.089	.08
14 S.G.	15	.243	.088	.41
	21	.222	.095	.26
	27	.110	.086	.06
	32	.100	.094	.02
15 N.G.	8	1.120	.117	1.92
	15	.540	.130	.73
	20	.278	.131	.25
	24	.186	.131	.10
	38	.139	.122	.02

TABLE 12, *continued.*

Case	Weeks	v_{tot} (mm)	v_t (mm)	s/E_f (s/E_fI_f)
16 Å.D.	15	.234	.094	.63
	27	.136	.093	.18
	33	.119	.086	.11
17 R.J.	7	.317	.122	.49
	12	.212	.121	.22
	16	.154	.117	.09
	21	.126	.117	.02
18 Å.B.	28	.452	—	.30
	33	.487	.107	.35
	37	.522	.117	.34
	43	.339	.124	.17
	55	.370	.111	.23
19 S.K.	23	.092	.068	.03
20 H.M.	17	.583	.145	.40
21 H.H.	15	*.235*	.137	*.29*
22 A.C.	14	.154	.105	.18
23 L.H.	14	.221	.085	
24 B.Ö.	—	.088	.054	
25 I.A.	22	2.030	.093	
	36	3.290	.086	
26 E.J.	22	.119	.090	(.05)
27 I.S.	18	.280	—	.28
	25	.238	.130	.22
	31	.192	.125	.13
	37	.181	.130	.09
28 V.G.	8	.524	.135	.86
	15	*.410*	.132	*.58*
	19	*.322*	—	*.40*
	27	.300	.141	.33
29 V.L.	31	.303	.077	.31
	41	.142	.074	.10
	46	.097	.073	.03
	51	.088	.075	.02
	55	—	.077	—
	75	.099	.088	.02
30 M.N.	28	.177	.123	.04
	39	.166	.124	.03
	48	.153	.117	.01
31 K.E.	13	1.290	.158	.54
	21	.760	.179	.29
	27	*.720*	.189	*.25*
	33	*.540*	.176	*.17*
	40	.389	.183	.11
32 U.K.	18	.154	.139	.03
	23	.163	.143	.04
33 C.S.	27	*1.240*	.167	*.67*
	38	*1.280*	.175	*.56*
	44	*.610*	.194	*.20*
	59	.349	.168	.11
34 K.Å.	15	1.220	—	.86
	39	.745	.159	.39
	45	*.560*	.168	*.28*
	52	.340	.165	.13
	61	.227	.162	.05
35 A.L.	—	.108	.092	.04
36 K.W.	29	*.265*	.124	*.10*
37 V.N.	13	*.360*	.175	
	20	*.335*	.225	
38 E.R.	17	.252	.143	(.15)
	22	.210	.130	(.11)
39 A.B.	—	.553	.096	
40 B.H.	16	*.400*	.155	*(.55)*

TABLE 13 *Values s/E_f and (s/E_fI_f) at various stages of treatment. The ranges are those between which treatment was changed, or the fracture was clinically stable. (Italic numbers are extrapolated values.)*

Case	Walking boot cast	Without cast: Two crutches	Without cast: One crutch	Fracture clinically stable	Full weight bearing
1	—.44		.44—.23	—.23	.23—.18
2	.36—.15			.36—.19	.15—.06
3	.27—.06			.27—.18	.18—.06
4	.29—.08			.29—.14	.08—.06
5	.94–.50	.50—	.22—	.50—.22	.22—.17
6		.21—		.21—.08	.21—.08
7	.79—			—.08	—.08
8	(.13—)			(—.13)	(.06—)
9		(.53—)	(.24—)	(—.53)	(—.12)
10	.85—	.15—		—.18	.15—.11
11		.36—	.29—	.29—.13	.23—.13
12	1.75—.30		.30—		—.07
13	.90—.19		.19—	.41—.19	.19—.08
14	.41—.06		.06—	.26—.06	.26—.06
15	1.92—.25		.25—	.25	.25—.10
16	—.18			—.18	.18—.11
17	.49—.09	.09—		.22—.09	.09—
18	—.35				.23—.17
20	.40—				
21		.61—			
22				—.41	.41—
27	.28—.13	.13—		.22—.13	.13—.09
28	*.58*—.40	.40—	.33—	.40—.33	
29	.30—.10		.10—	—.03	.03—
31		.29—	*.25*—	.29—.25	*.17*—.11
33	*.56*—.20	.20—		.20—.11	.11—
34	*.28*—.13		.13—	—.13	.13—.05
36				—*.10*	
38		(.15—)			
40	(*.55*—)				

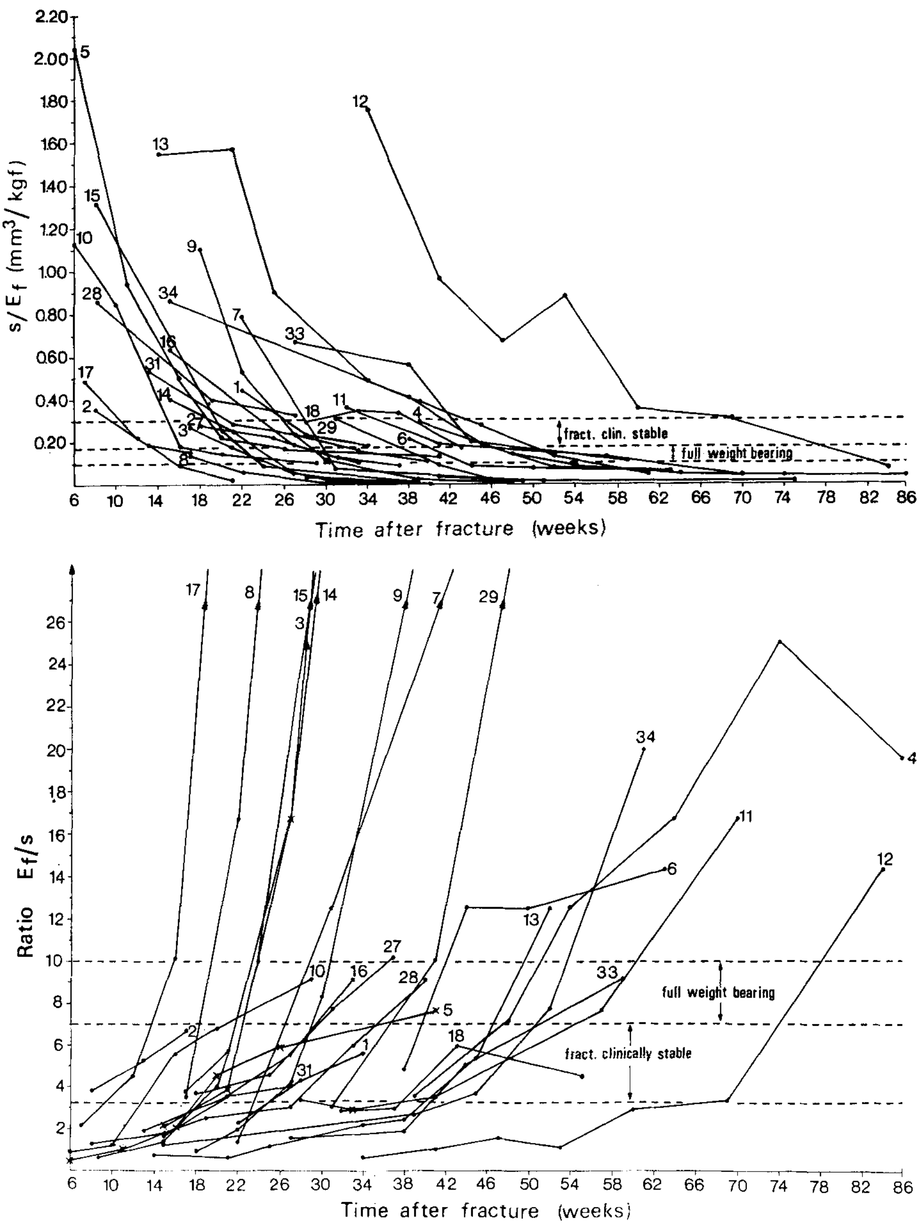


FIGURE 20 *Upper.—The ratio s/E$_f$ as a function of time.*
Lower.—The inverse ratio (E$_f$/s) as a function of time.
The slope of the curves for the various cases indicates the rate of healing, expressed as a decrease in specific deflection and as an increase in stiffness of the callus, respectively.

The rate of healing can also be expressed in terms of the increase in the stiffness of the callus with time. When the deflection within the callus tends to zero—that is to say, when the deflection of the fragments accounts for nearly the whole of the total deflection of the tibia—E_f tends to high values. Curves of E_f/s as a function of time are shown in figure 20.

The limits within which full weight bearing was started or clinical stability of the fracture was first noted are given in the two figures.

In four cases, 4, 32, 33 and 37, a slight increase in deflection was found to have occurred between two examinations (Table 12). In three of these cases the measurements in question were performed after full weight bearing had begun, and at most a small reduction in deflection would be expected; however, the values do not differ by more than can be ascribed to the error of the method. In the fourth case, no. 33, the explanation may be the same, although here it is at the second measurement that the increase was recorded. It could than have its explanation in extremely slow healing in the interval; but, again it might also be due to the random error of the method.

At three examinations an increase of more than 10 per cent was recorded. In case 12 the deflection was 25 per cent greater after 53 weeks than after 47; this increase must probably be ascribed to a reduction in the stability of the fracture. The long period of immobilization had resulted in stiffness of the knee and ankle and the cast was therefore removed and physiotherapy given. When, after the fifty-third week, a knee cast had been furnished and weight could be put on leg, the stability increased.

In case 18 the corrective osteotomy was followed by delayed union. The second, third and fifth values showed increases in deflection of 8, 7 and 9 per cent, whereas the fourth value indicated a reduction of 35 per cent. Here, too, the figures probably represent the true state of affairs; for during the first three measurements and between the fourth and the fifth there was no signs of progress in healing and it is probable that a pseudarthrosis was in the course of formation. For some reason—perhaps an increase in the load on the transverse fracture while the PTB cast was being worn —signs of stabilization were noted between the third and the fourth measurements. No further measurements of the deflection have yet been made.

In case 25 the deflection measurements were followed immediately by operations to hasten healing in the extremely mobile fracture, with a consequent change in the experimental conditions between the two measurements.

VII. *Discussion*

ACCURACY OF THE METHOD

The observed deflection of the bone contains an error that is dependent on the reliability of the measuring instrument. A further error is that inherent in the principle of the method, which is strictly applicable to a simple beam, whereas here we are concerned with a body of irregular shape and structure, composed, moreover, of biological tissue.

The total measurement error of the registration unit was ± 1.2 per cent. There is a systematic error of the method by virtue of the bending of the instrument and the local resilience of the bone around the pins; this error was calculated at 39 and 29 μm at pin distances of 24 and 21 cm, respectively. To obtain an impression of this local resilience measurements were made of the displacement in various materials, namely, pieces of spruce wood, ebonite, oak and laminated bakelite. The respective displacements were 36, 27, 21 and 10 μm. It may be inferred that the resilience of the pins is dependent to some extent on the physical properties of the various bones. This means that the application of a constant correction will increase the deviation of the values obtained in the calculation of the modulus of elasticity.

It is difficult to obtain a secure hold for the pin tips in bone with a thin cortex. Two attempts were made to replace the ordinary pins that had perforated the cortex with ones having threaded tips, which would be expected to give a better hold in the cortex. The values then obtained were the same as those values extrapolated from the curve.

The variation in the shape of the tibia and its heterogenous structure affect the determinations of both deflection and moment of inertia. From a visual estimate of the direction of the principal central axes of the moment of inertia of the section at different levels on 10 specimens and from an integrator determination in one specimen (Fig. 3) it would appear that these axes are practically parallel. It is only in the condylar area and near the malleolus that the principal central axes change direction; here the cross section is closer to a circle and, since the difference between the maximum and the minimum moments of inertia of the section is small, the directions of the axes are of relatively little importance. The anterior crest has a sigmoid course and, as can be seen in figure 3, it twice intersects the plane of the principal axes. On section +12 the tuberosity of the tibia lies on the medial side of the principal central axes and on section +6 the crista anterior just passes across to the lateral side and then, in section —6, returns to the medial side. Thus, the torsion due to any twisting of the tibia's bending plane for the minimum moment of inertia may be neglected. On the other hand, the angle between the bending plane of the instrument and that for the minimum moment of inertia of the tibia when the longitudinal axes are parallel can decrease the observed deflection by up to 3 per cent. When the bending planes intersect and when they are translated and the longitudinal axes of the instrument and the tibia are kept mutually parallel, the deflection due to pure bending is increased by a torsional component; this amounts to about 2 per cent of the observed deflection.

Of greater importance, however, is the position of the middle pin. This should be the same for different measurements on the same bone, but this ideal is not always achieved. One of the reasons why a change in level of the middle pin has taken place is the difficulty of determining the contour of the medial malleolus, especially when the ankle is swollen. An attempt was made to retain the marking from one occasion to the next in order more easily to identify the sites for the pins. The most effective procedure was to fill in the old marks after the cast had been removed and before the bone was sterilized.

The deflection of the fractured tibia would presumably be affected by stress in the surrounding tissues, by any swelling of the leg and by the fluid content of the bone tissue (Sedlin & Hirsch, 1966). Opinions differ on whether the bone might be hydraulically strengthened, through the effect of the blood pressure or muscle contraction (McPherson & Juhasz, 1965; Swanson & Freeman, 1966). As, however, the significance of these factors is small and probably essentially the same for the different tibias it may be disregarded in the evaluation of the stability of the fracture.

The determination of the moment of inertia from eqn. [8] incurred a small measurement error, and the reproducibility was satisfactory. If the variation of the moment of inertia along the tibia is taken into account an even more reliable expression for the modulus of elasticity for a single observation is obtained. This was done by calculating the moments of inertia from other approximation formulae as eqn. [8] at different levels of the tibia, thereby obtaining an individual and more correct constant in eqn. [9].

The ratio s/E_f is estimated on the basis of several assumptions and approximations. The line of fracture varies and the length of the callus, the modulus of elasticity and the moment of inertia of the callus are none of them properly defined measures; s and E_f are used solely as an aid in determining the additional deflection caused by the callus of the fractured bone, and insofar as the callus cannot be regarded as a short beam or spring joint, these two terms cannot be considered to represent the real length of the callus or its modulus of elasticity.

In the mathematical model the viscous element, its variation with time, and the shear in the callus are disregarded. Nor is it entirely correct to assume that the recorded deflection of the bone fragments are identical with that of the sound tibia v_t having the same systematic error of measurement for the fractured and the sound tibia (see eqn. [12]).

Because of metabolic and circulatory changes, however, this would result in a smaller value of δ_f. Experiments on laboratory animals have shown that the healing of fractured bone is accompanied by changes in the physical properties of bone substance. The resistance of the tibia condyle to compression has been found to be lower for the bone with the fractured shaft (Nilsson & Smith, 1969).

Any variation in the ratio s/E_f is due chiefly to a change in the value of E_f. According to Henry *et al.* (1968) there is a linear relationship between the stiffness and the breaking strength of the damaged leg. In bending experiments on the rabbit in which one radius had been fractured surgically, these authors showed that during the course of healing the ratio between the ultimate breaking strengths of the fractured and sound radii varied directly as the ratio between the stiffnesses of the fractured and sound radii. The slope of the line was 0.48; that is to say, when the stiffness ratio was unity the strength ratio was about 0.5.

This would mean that when the recorded deflections of the fractured and the intact tibia are equal, the ultimate bending strength of the callus is half of the intact tibia at the same level. In the experiments on rat tibia, Wray & Goodman (1963) found that the breaking strength 65 days after the surgically produced fracture was greater for the fractured than the sound tibia, and after the 80th day the difference was statistically significant.

From the curves of s/E_f as a function of time in figure 20 (upper) it is seen that the specific deflection of the callus during the course of healing decreased at a varying rate. The curves of E_f/s as a function of time show that the increase in stiffness of the callus also occurs at a varying rate (Fig. 20, lower part). According to the study by Henry *et al.*, when there is no deflection of callus, that is, when $\delta_f = 0$, its ultimate breaking strength is one half that of the sound tibia bone at the same level. This would mean that up to the breaking point the callus becomes more resistent to bending forces; but there is no deflection of the fractured tibia through any weakening at the fracture site. Thus, during the further healing of the fracture the ultimate breaking strength will increase while the flexural rigidity remains constant. Clinicians consider that in general a fracture is healed, when the stiffness has been recovered. However, as the mineralization of the callus is then still incomplete it would appear that the strength goes on increasing until osseous consolidation is complete and the bone structure fully restored.

CLINICAL VALUE OF THE METHOD

In cases where union is delayed or pseudarthrosis is suspected it is necessary to examine whether healing can be expected at all. Just what is to be gained from an operation to hasten the process is not always obvious. Most workers assess the indications for surgery on the basis of the time that has elapsed since the fracture occurred, but a long interval does not necessarily mean that the development of the stability of the fracture has stopped and it is this that should decide whether an operation should be performed. Without an objective knowledge of the stability of the fracture, it is impossible to decide whether an operation is indicated. The method reported not only provides the means for resolving this dilemma; it also enables an objective evaluation of different forms of treatment to be made.

An experienced investigator requires about 30 minutes to perform a determination of the deflection on both legs, with the accompanying radiographic examination. To appraise the development of the stability in a single case, however, it is enough to measure the deflection of the fractured bone under the same conditions two or three times over a convenient period, and each measurement will require only 10 minutes or so. In over 900 pin insertions there were no signs of infection or subsequent irritation, or of any other complications. The examinations were performed under local anaesthetic and are painless.

The bending test technique can be applied to other shaft fractures, such as those of the femur, but then another, longer, measuring beam has been used. It may also be possible to use the method for diagnostic purposes in diseases affecting bone.

CONCLUSIONS

The bending of the tibia *intra vitam* under a force of 5 kgf can be determined in a single measurement to an accuracy of 5 per cent for men and 8 per cent for women.

The minimum moment of inertia of the tibia can be found in a single measurement to an accuracy of 5 per cent.

An *intra vitam* determination of the modulus of elasticity of the tibia can be made in a single measurement to an accuracy of 10 per cent, while for young males in sound health the error incurred in the measurement of the modulus of elasticity was 5 per cent.

The values obtained for the modulus of elasticity are probably on the high side because it was impossible to determine the moment of inertia in spongy bone.

The method is intended primarily for patients with fractures of the tibia shaft. Of the 133 measurements performed on fractured tibias there were only 3 where a straightforward interpretation of the value could not be made.

VIII. Summary

The object of this investigation was to devise a method for obtaining an objective measure of the stability of a fracture of the tibia shaft during the course of healing.

An instrument has been designed with which the deflection of the bone can be registered as a function of the applied force.

The deflection of a bone is dependent on the moment of inertia of the section and the modulus of elasticity. A procedure has been evolved for determining the moment of inertia from a radiograph. It is thus possible to compare the deflection of various bones in terms of the varying modulus of elasticity in bending.

The accuracy of the method was examined on 20 pairs of autopsy specimens in 40 patients with tibial shaft fractures and in 9 voluntary subjects.

During the course of healing the gap between the fragments is filled with callus, whose physical properties gradually change towards those of bone. This implies a decrease in the deflection due to the weakness of the callus. As healing progresses the stiffness of the callus increases. On the basis of the values obtained from measurements carried out with the described method an expression has been deduced by means of which it is possible to compare the degree of healing in terms of quantities that are equivalent to the stiffness of fracture callus.

The clinical series consisted of 26 men and 14 women with a fracture of the tibia shaft; in most of them union was slow. The first stability measurement with the method was made, on average, 19 weeks after the fracture, and determinations were continued until full loading of the fractured leg was possible—on average 33 weeks after the fracture.

Altogether 133 such determinations were performed on fractured tibias; in only three could no definitive conclusions be drawn as regards the development of stability.

The clinical application of the method for examination of the fracture was instructive and facilitated the evaluation of the course of healing. The measurements were performed ambulatory under local anaesthetic. In more

than 900 cases there was no infection or other complication. The examination took on average 30 minutes.

In the case of delayed union the appraisal of the stability of the fracture by means of the method can be of value in deciding whether to resort to an operation in order to hasten union.

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