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GEOMETRIC PROBLEMS OF OSTEOTOMY

With special regard to rotation-angulation osteotomy

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In the following are considered the geometric problems in osteotomy where rotation and angulation are combined.

This problem may, for example, exist in a patient where a femoral shaft fracture has healed with abbreviation and rotation deformity. It may also exist in cases of slipped femoral epiphysis. In placing the osteotomy in the sub- or intertrochanteric region, varisation or valgisation and rotational correction can be achieved. Ball and socket osteotomy has been recommended for this purpose (Campbell 1971). Southwick (1967) described a method where angulation in the a.–p. plane and the side projection were combined. However, the adaptation between the two bone-cut surfaces in the described methods may be very poor with delayed healing as the result.

An ideal adaptation with the greatest possible contact surface between the bone fragments is only obtained when the osteotomy is held within one plane and the displacement is made in this plane.

If a cylindric stick is cut right-angled to the centerline and the one fragment is turned around this line, no angulation between the centerlines of the two fragments will take place.

If the angle between the cut-plane and the centerline, i.e. the cut-inclination (Θ in Figure 1), is different from 90° and we rotate the two fragments in relation to each other around an axis vertical to the cut-plane, the centerlines of the fragments no longer will be the continuations of each other. They will determine an angle, the angulation (Ψ in Figure 2). This angle will increase with the degree of rotation. It will also increase the more the cut-inclination differs from 90°, i.e. the smaller the cut-inclination. The correlation between the cut-inclination, in cases of osteotomy called the osteotomy-inclination, the rotation of the cylindric stick, i.e. the bone, and the developed angulation is calculated below and tabulated in Table 1.

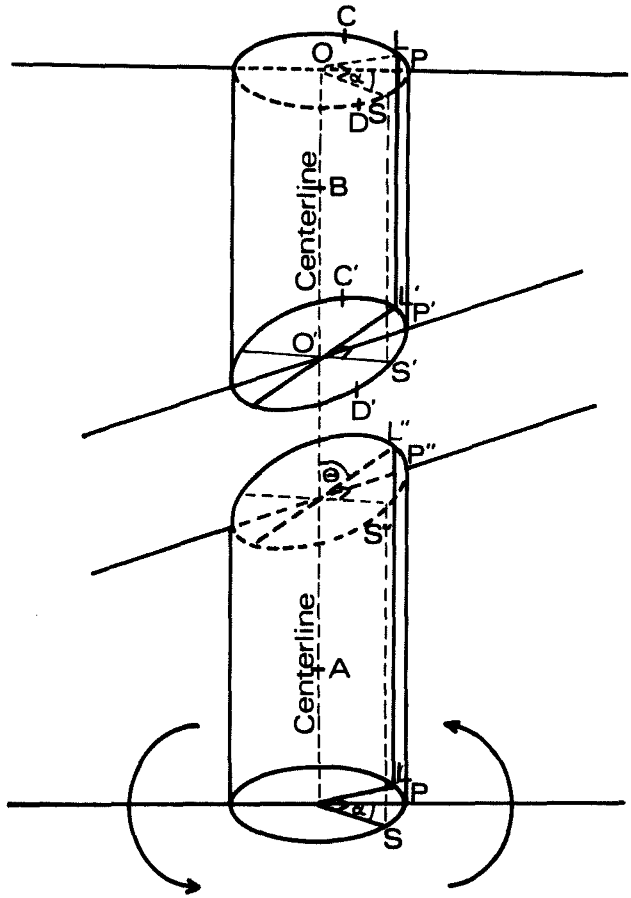


Figure 1. A cylindric stick (femur) is cut with a cut-inclination (osteotomy-inclination) Θ. The cut surfaces are ellipses, the long axes of which are projected to the (right side of the) surface of the stick in L' and L'', the short in S' and S''. The axes are right-angled to each other. The planes containing the centerline and the long and the short axis respectively are projected to the surface of the stick in the lines L-L and S-S respectively. P P'P''P represents the "paper plane". The angle α is half the wanted rotation. For explanation of other letters see text.

A SKETCH OF THE MATHEMATICAL CALCULATIONS

The object of these calculations is to define the correlation between the angulation, the rotation and the osteotomy-inclination.

First we introduce our notation: The variables described above, angulation, rotation and osteotomy-angle, are denoted Ψ, γ and Θ respectively. It is necessary to introduce a new variable, the cut-plane-rotation φ, which is the angle between the two long axes of the ellipsiform cut-surfaces after the osteotomy. This new variable

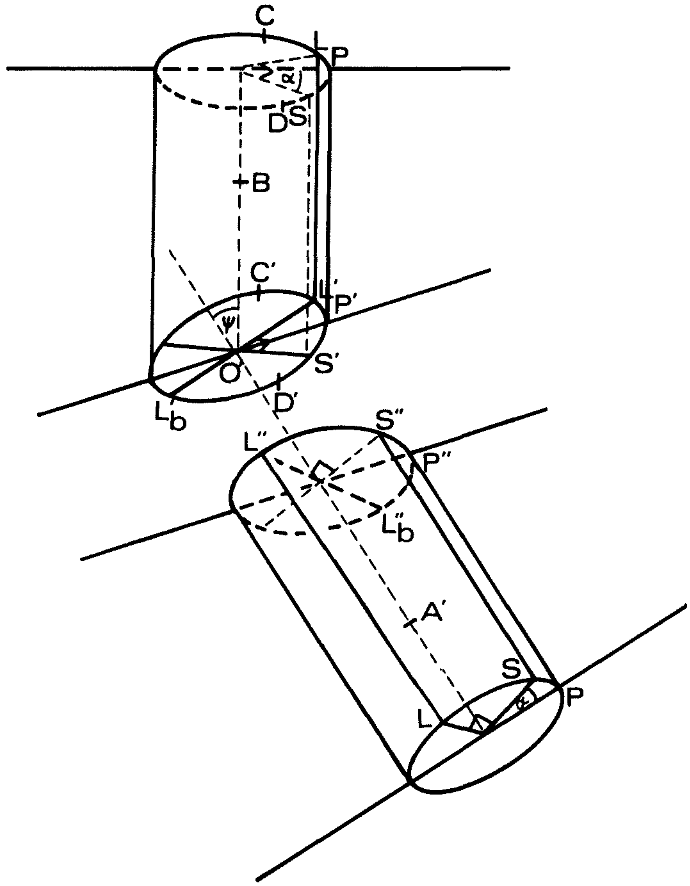


Figure 2. The lower fragment is rotated the angle 2 α. The centerlines of the two fragments are contained within the "centerline plane" PP'P"P. The long axis of the cut-surfaces are marked L'L_b and L" L"$_b$ respectively. The angulation between the fragments, i.e. between the centerlines, is Ψ.

will facilitate the calculations. At first the correlation between Ψ, φ and Θ should be found.

Consider the points O', A and B, where O' is the center of the cut-surface, and A and B are placed in the distance 1 (one) from O' on each centerline before rotation (Figure 1). These points should be regarded as in a three-dimensional coordinate system with origo O' and the centerline lying in the plane y = o and the cut-surface lying in the plane z = o. Here A and B will have the coordinates

$$A = \begin{pmatrix} \cos\Theta \\ o \\ \sin\Theta \end{pmatrix} \qquad B = \begin{pmatrix} -\cos\Theta \\ o \\ -\sin\Theta \end{pmatrix}$$

Table 1. The osteotomy-inclination is tabulated as a function of the wanted angulation (above) and the wanted rotation (left). The table is based on calculations made on a computer. For further explanation see text.

Rotation	Angulation 5	10	15	20	25	30	35	40	45	50	55	60
5	—	—	—	—	—	—	—	—	—	—	—	—
10	46	—	—	—	—	—	—	—	—	—	—	—
15	69	—	—	—	—	—	—	—	—	—	—	—
20	75	47	—	—	—	—	—	—	—	—	—	—
25	78	63	—	—	—	—	—	—	—	—	—	—
30	80	69	48	—	—	—	—	—	—	—	—	—
35	82	72	60	—	—	—	—	—	—	—	—	—
40	82	75	65	49	—	—	—	—	—	—	—	—
45	83	77	69	59	—	—	—	—	—	—	—	—
50	84	78	71	63	50	—	—	—	—	—	—	—
55	85	79	73	66	57	—	—	—	—	—	—	—
60	85	80	75	68	62	50	—	—	—	—	—	—
65	85	80	75	70	64	57	—	—	—	—	—	—
70	85	81	77	72	66	60	51	—	—	—	—	—
75	86	82	77	73	68	62	56	—	—	—	—	—
80	86	82	78	74	70	65	59	52	—	—	—	—
85	86	82	79	75	71	66	61	55	46	—	—	—
90	87	83	79	75	72	67	63	58	52	—	—	—
95	87	83	80	76	72	69	65	60	55	47	—	—
100	87	83	80	77	73	70	66	62	57	52	43	—
105	87	84	80	77	74	70	67	63	59	55	49	—
110	87	84	81	77	75	71	68	65	60	57	52	45
115	87	84	81	78	75	72	69	65	62	58	54	50
120	87	84	81	78	75	72	70	66	63	60	56	52
125	88	85	82	79	76	73	70	67	64	60	57	53
130	88	85	82	79	76	73	70	67	65	62	58	55
135	88	85	82	79	76	74	71	68	65	62	59	56
140	88	85	82	80	77	74	71	68	66	63	60	57
145	88	85	82	80	77	74	72	69	66	63	60	57
150	88	85	82	80	77	75	72	69	67	64	61	58
155	88	85	82	80	77	75	72	70	67	64	62	59
160	88	85	82	80	77	75	72	70	67	65	62	60
165	88	85	82	80	77	75	72	70	67	65	62	60
170	88	85	82	80	77	75	72	70	67	65	62	60
175	88	85	82	80	77	75	72	70	67	65	62	60
180	88	85	82	80	77	75	72	70	67	65	62	60

The rotation is now performed by rotating A the angle φ around the z-axis to A' (Figure 3), where

$$A' = \begin{pmatrix} \cos\Theta \cos\varphi \\ \cos\Theta \sin\varphi \\ \sin\Theta \end{pmatrix}$$

The area of the parallelogram stretched by A'O'B can then be determined as the length of the vectorial product of $\overline{O'A'}$ and $\overline{O'B}$, i.e.

$$| O'A' \times O'B | = \left| \begin{pmatrix} -\cos\Theta \\ 0 \\ -\sin\Theta \end{pmatrix} \times \begin{pmatrix} \cos\Theta \cos\varphi \\ \cos\Theta \sin\varphi \\ \sin\Theta \end{pmatrix} \right|$$

$$= \sqrt{\cos^2\Theta \ (\sin^2\varphi + \sin^2\Theta \ (\cos\varphi - 1)^2)}$$

But the same area is determined by $\sin\Psi$, hence we get

1. $\sin^2\Psi = \cos^2\Theta \ (\sin^2\varphi + \sin^2\Theta \ (\cos\varphi - 1)^2)$

However, it was not the cut-surface-rotation φ that should have our attention, but the final rotation γ as expressed in relation to the centerline-plane, i.e. the plane containing both centerline-fragments after the angulation; hence φ should be eliminated from the equation by inserting γ. The relationship of φ and γ is illustrated in Figure 1. In this figure φ is the angle C'O'D' and γ is the angle COD. Regarding the coordinates in what could be called the natural coordinate-system, we have OC and OD with the coordinates

$$\begin{pmatrix} 0 \\ \mp \sin \dfrac{\gamma}{2} \\ \cos \dfrac{\gamma}{2} \end{pmatrix}$$

and correspondingly O'C' and O'D' with the coordinates

$$\begin{pmatrix} \cos \dfrac{\gamma}{2} \ \cot\Theta \\ \mp \sin \dfrac{\gamma}{2} \\ \cos \dfrac{\gamma}{2} \end{pmatrix}$$

From this we get

$$\sin\varphi = \frac{| O'C' \times O'D' |}{| O'C' | \cdot | O'D' |} = \frac{2 \sin \dfrac{\gamma}{2} \ \cos \dfrac{\gamma}{2} \ \sin\Theta}{(\sin^2\Theta + \cos^2 \dfrac{\gamma}{2} \ \cos^2\Theta)}$$

and

$$\cos \varphi = \frac{|O'C' \times O'D'|}{|O'C'| \cdot |O'D'|} = \frac{\cos^2 \frac{\gamma}{2} - \sin^2 \frac{\gamma}{2} \sin^2 \theta}{(\sin^2 \theta + \cos^2 \frac{\gamma}{2} \cos^2 \theta)}$$

By inserting this in equation 1, we find the correlation we have been looking for. Even after simplification by reductions, it is too complicated for manual calculations. Hence we present the correlation in form of Table 1 after calculation on a computer. This table may also be established (or proved) from experiments with a model.

After having determined the level for the osteotomy the problem is to determine the orientation of the cut-plane around the centerline. The centerlines of the two fragments determine after rotation a certain

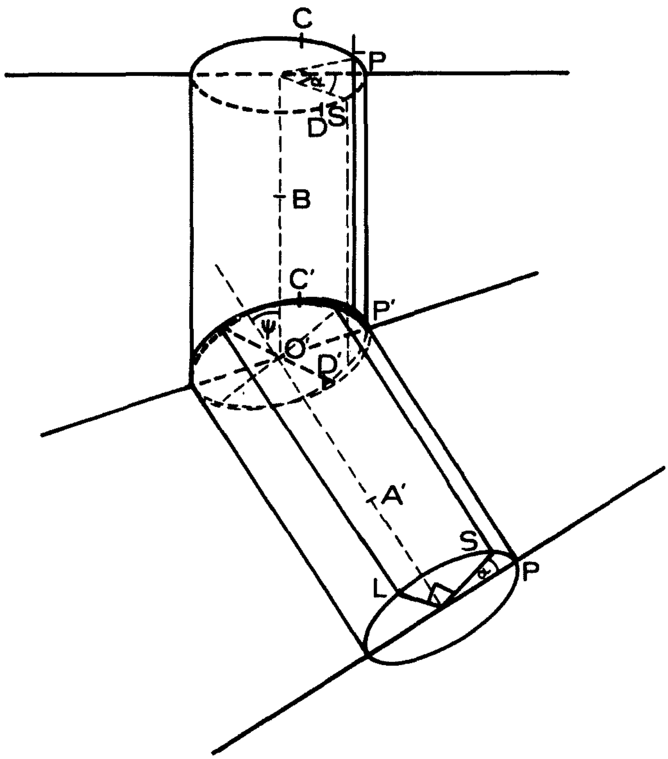


Figure 3. The cut-surfaces are brought in broad contact keeping the centerlines within the PP'P-plane (centerline-plane). Other letters: see text of "A sketch of the mathematical calculations".

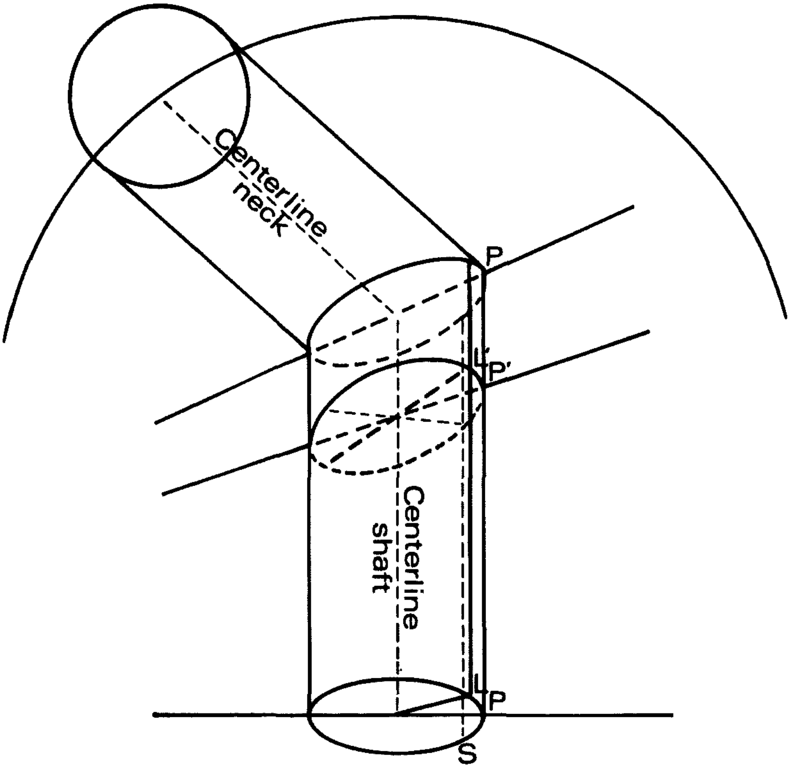


Figure 4. A "model" of the upper end of the femur. The centerlines of the femoral neck and of the femoral shaft are kept within the centerline-plane PP'P ("paper plane"). Osteotomy is marked with the long axis of the osteotomy-surface projected to L'.

plane, the centerline-plane, in which the angulation takes place. This plane (PP'P"P in Figures 1–3) divides in two equal angles the angle between the short axes of the cut-surfaces. Using this fact it is possible to plan the osteotomy.

First the centerline-plane must be fixed. The projection of this plane on the outer surface of the *upper* fragment is outlined, P-P in Figure 1, and the angle α (= half the wanted rotation) is marked back from this line in relation to the turning direction for the *lower* fragment (in Figure 1 towards the viewer). A line S-S is outlined. The ellipsiform cut-surface can now be characterized by having its short axis in the plane determined by the centerline and the line S-S, right-angled to the centerline at the level where the osteotomy is wanted. Marking 90° around the cylindric surface (i.e. around the centerline) from S-S, the

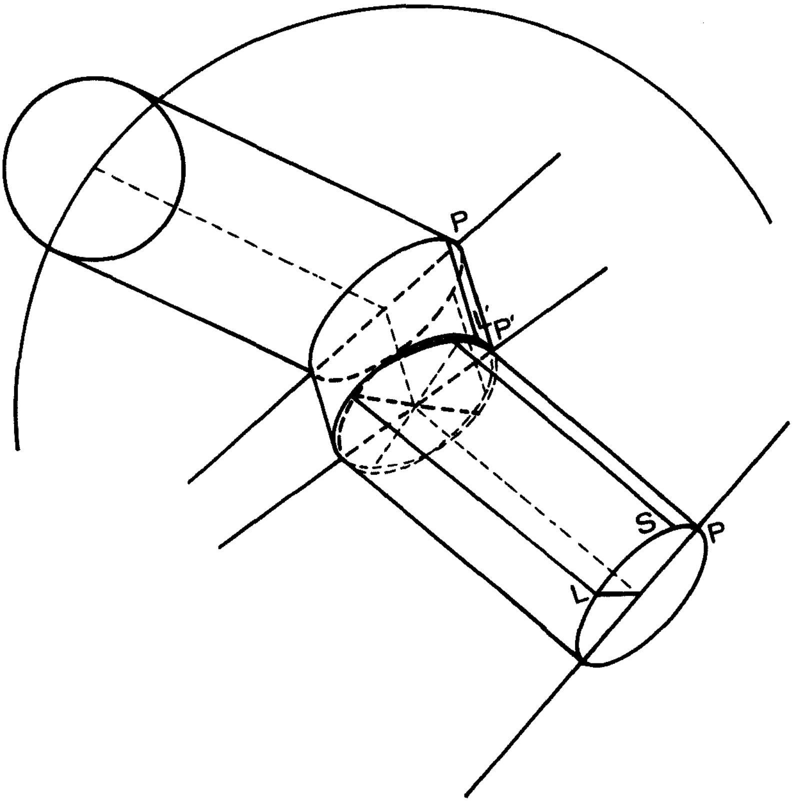


Figure 5. Rotation has been performed as in Figure 4. The centerlines are still in the plane PP'P.

line L-L is outlined. The long axis of the cut-surface is contained in the plane determined by the centerline and L-L. The size of the angle between the centerline and the long axis of the cut-surface is the osteotomy-inclination Θ. This is found in Table 1 knowing the wanted angulation and rotation after the perfected osteotomy.

For each osteotomy-inclination there are two possibilities for the osteotomy-plane. They have the same short axis, while their long axes are arranged symmetrically around the centerline. In performing the osteotomy one should consider that the final apex of the angulation between the two fragments will be exactly in the middle between the summits of the osteotomy-surfaces of the two fragments (in Figure 2 between the ends of the long axes marked L'_b and L").

In cases of osteotomy in close relation to the trochanteric region, maximal elongation of the femur is obtained when the centerline of the

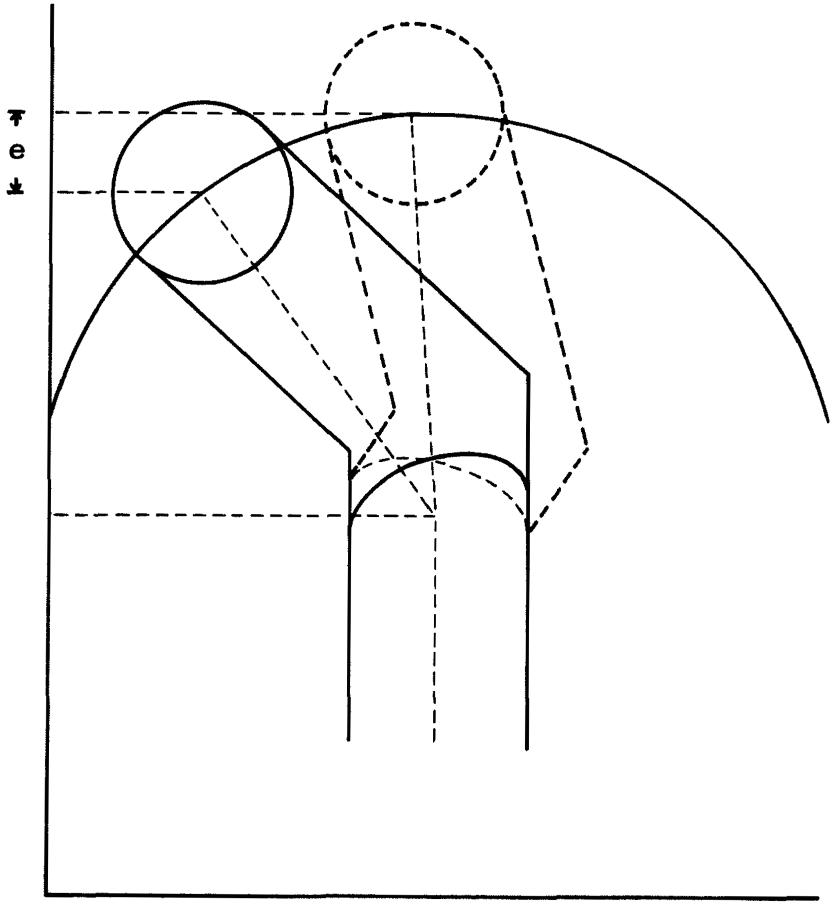


Figure 6. The elongation of the femur obtained by a subtrochanteric valgisation oesteotomy may be calculated from a drawing like this.

femoral neck and the two shaft-fragments are kept within the same plane (Figures 4 and 5). The elongation can be determined from the drawing Figure 6. If another direction of the collum-part is wanted the angulation-plane should be placed differently. For example, in cases of slipped femoral epiphysis a forward tilting of the femoral head is wanted together with a valgisation of the femoral neck. This can be obtained by placing the angulation-plane, i.e. the centerline-plane, with the angulation "open" forward and laterally.

To illustrate how the principles of the outlined method can be used, even without Table 1, the following case is reported.

Figure 7. X-ray of the femur from the reported case. A shaft fracture has healed with 40° outward rotation of the distal fragment and 3½ cm abbreviation (March 71).

CASE REPORT

In November 1969 a 24-year-old man sustained a fracture in his left femoral shaft in a car accident. He was admitted to hospital and treated by insertion of a Küntscher medullary nail and cerclage wire. Unfortunately the healing occurred with some shortening of the leg and outward rotation of the foot (Figure 7).

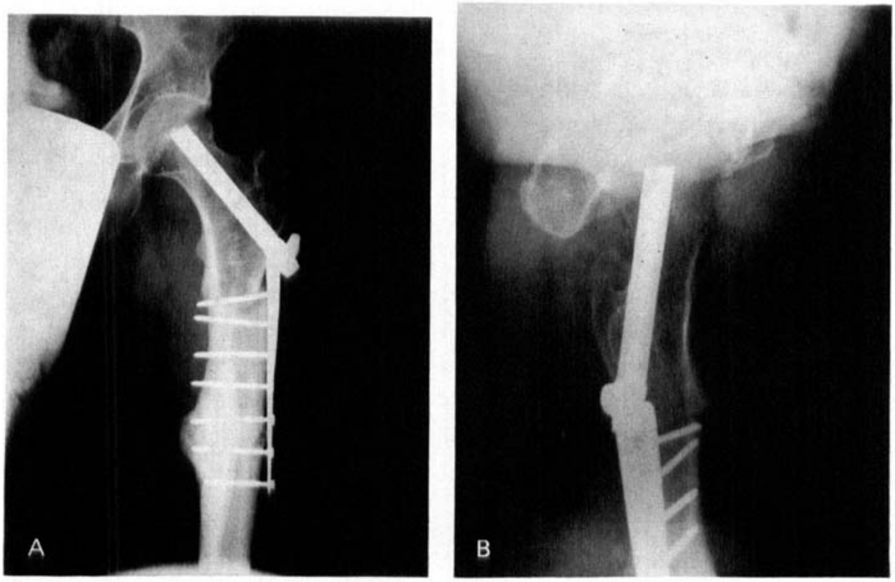
In August 1971 the patient was admitted to the County Hospital of Arhus for correctional operation. A 3½ cm shortening of the left leg was found. The left foot deviated outwards with an internal rotational defect of 15° and an external rotation of 100°. X-ray examination revealed an external rotation on the fracture site of 40°.

The Küntscher nail was removed and a subtrochanteric osteotomy with derotation and valgisation was performed (Figure 8 A–B). The location for the osteotomy and the osteotomy-angle were determined by cutting a model made from a wooden stick (a brush-shaft). The principles outlined above were used, but Table 1 was not as yet available. There was good broad contact on the osteotomy-site and it healed quickly. The patient began weight-bearing after 7 weeks.

In February 1973 the internal rotation of the left hip was 10° and the external rotation 70°. With the patient supine there was no difference in the position of the two feet. With the patient standing a 2½ cm shortening of the left leg was found.

DISCUSSION

In the referred case the rotational correction was satisfactory, but the elongation was only around 1 cm of the missing 3½ cm. Comparing the X-rays (Figures 7 and 8 A) it is seen that the obtained angulation



Figures 8 A – B. Same case as Figure 7. A subtrochanteric osteotomy has been performed with 30° inward rotation and 10° valgisation, giving 1 cm elongation of the femur (August 71).

is around 10°. The maximal elongation would have been obtained if the center of the femoral head had been turned right up in the elongation of the centerline for the femoral shaft, i.e. with an angulation of around 23°. However, the maximal elongation would only have been around 1.5 cm compared to the obtained 1 cm, and the last few extra millimeters had to be balanced against a much less acceptable position in the hip and probably more difficult osteosynthesis.

In planning an osteotomy of the actual kind it must be emphasized that it is easier and safer as the first step to make a model from a wooden stick or any cylindric shaped object, which can easily be cut in different angles. The more detailed determination of the osteotomy-location and -angulation must be determined from the wanted effect on rotation, angulation and hereby elongation. The further down below the intertrochanteric region the osteotomy is placed the lesser is the elongation effect.

Whether it should be preferred to determine the osteotomy-inclination by the calculations in Table 1 or by means of experiments with a model made from a wooden stick is up to the surgeon.

SUMMARY

The problems of osteotomy where rotation and angulation and even elongation are wanted, are treated clinically and mathematically. The work is especially concerned in osteotomies in the trochanteric region, where an angulation will be able to change the steepness of the femoral neck.

Instructions for a simple osteotomy making alterations possible in all directions are outlined in detail. The outlined method gives the best possible conditions for healing of the osteotomy.

By means of a computer-calculated table the osteotomy-inclination can be determined, when the desired angulation and rotation is known.

Finally, a case is reported where a femoral shaft fracture had healed with shortening and outward rotation of the foot. It was treated by the described method.

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