

Electromagnetic and electric field configurations produced by two coils

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1. Introduction

Presently the specific electric parameters acting under electromagnetic stimulation of bone is unknown.

A first approach to this problem is to determine the topographic configuration of the different possible parameters in the space limited by the coils of the stimulator : local electric field, intensity of electric current, local induction field...

Our aim is to compute the values of the induction field and of the electric field produced by two coils. The shape of the coils are parametrically defined. The computed results are experimentally verified.

11. Theoretic analysis

Induction field and electric field are computed using Maxwell's relations, involving the vector potential \vec{A} .

Concerning our specific problem we have :

$$\text{induction field } \vec{B} = \text{curl } \vec{A} \quad (1)$$

$$\text{electric field } \vec{E} = - \frac{\partial \vec{A}}{\partial t} \quad (2)$$

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The vector potential \vec{A} is defined by the following relation :

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{Id\vec{l}}{R} \quad (3)$$

The different parameters are represented on figure 1.

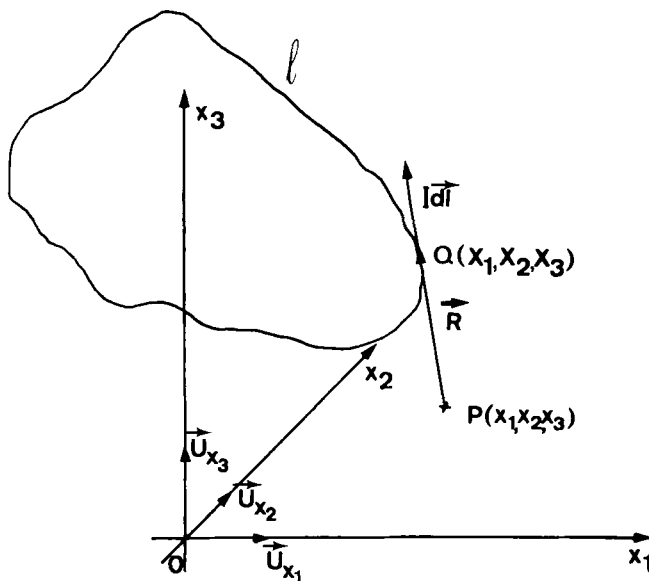


FIG. 1. — l is a closed loop crossed by the current I . $Q(x_1, x_2, x_3)$ is a point of the loop. $P(x_1, x_2, x_3)$ is a point where vector potential \vec{A} , electric field \vec{E} and induction field \vec{B} are to be calculated.

$$\vec{R} = (X_1 - x_1)\vec{u}_{x_1} + (X_2 - x_2)\vec{u}_{x_2} + (X_3 - x_3)\vec{u}_{x_3}$$

We compute the vector potential in a medium having the same magnetic properties as vacuum. This approximation is true for the biological tissues.

We suppose the biological tissue slightly conductive : so the contribution of induced currents in the tissues may be neglected compared to the current in the magnetizing loops of the stimulator.

We can define the co-ordinates of any point, Q , of the magnetizing loop(s) as follow :

$$x_1 = x_1(\alpha) \quad x_2 = x_2(\alpha) \quad x_3 = x_3(\alpha) \quad (4)$$

where α is a parameter.

So we find the components of vector potential A :

$$A_i = \frac{\mu_o I}{4\pi} \int_{\alpha_o}^{\alpha_f} \left(\frac{1}{R(x_1, x_2, x_3, \alpha)} \frac{\partial X_i}{\partial \alpha} \right) d\alpha \quad (5)$$

$i = 1, 2, 3.$

Relation (1) allows to compute the components of the induction field :

$$B_i = \frac{\partial A_k}{\partial x_j} - \frac{\partial A_j}{\partial x_k} \quad (6)$$

$i = 1, 2, 3$

and $|B| = \sqrt{\Sigma B_i^2}$

Relation (2) gives :

$$E_i = -\frac{\mu_o}{4} \frac{\partial I}{\partial t} \int_{\alpha_o}^{\alpha_f} \left(\frac{1}{R(x_1, x_2, x_3, \alpha)} \frac{\partial X_i}{\partial \alpha} \right) d\alpha \quad (7)$$

$i = 1, 2, 3$

and $|E| = \sqrt{\Sigma E_i^2}$

Practically we computed $\frac{|B|}{I}$ and $\frac{|E|}{\frac{\partial I}{\partial t}}$ which are expressions inde-

pendent of the absolute dimensions of the stimulator, the waveform and the amplitude of the magnetizing current.

The value « a » represents a length characterizing the stimulator. We carried out the calculations in two particular cases :

a) two circular coils situated in two parallel planes :

$$x_1 = a \cos \alpha \quad x_2 = a \sin \alpha \quad x_3 = \pm z_o \quad (\text{fig. 2})$$

b) two coils approaching the form of a clinical stimulator :

$$x_1 = a \cos \alpha \quad x_2 = b \sin \alpha \quad x_3 = \pm (z_o - c |\cos \alpha|) \quad (\text{fig. 3})$$

III. Experimental measurements

The induction field is measured by means of a little test coil composed of eight hundred loops and connected to an oscilloscope. The electric field is measured only in the case of the simplified stimulator (a) with the aid of a set of auxiliary loops of different radius connected to an oscilloscope. Measurements of total induced electromotive force in such loops allow computation of the local electric field in this case.

As results are independent of the waveform of the magnetizing current we performed measurements with alternating current.

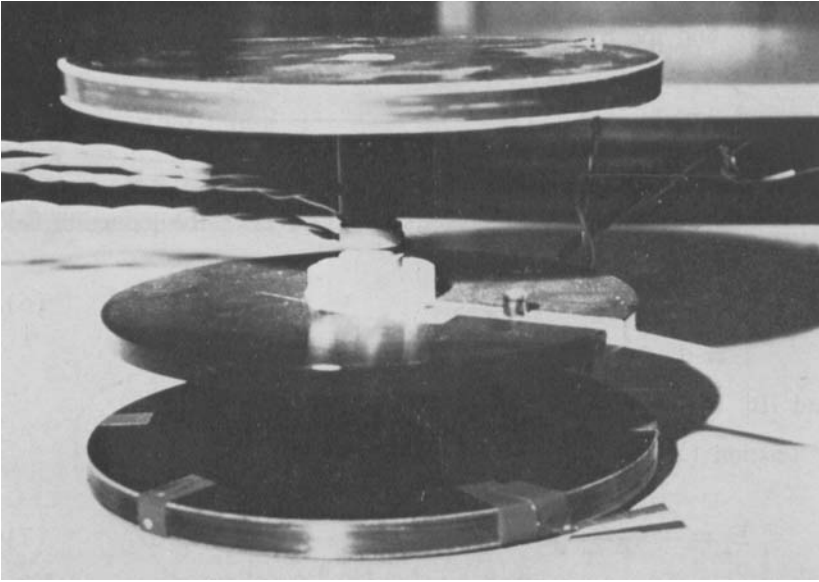


FIG. 2. — Stimulator composed of two circular coils in parallel planes.

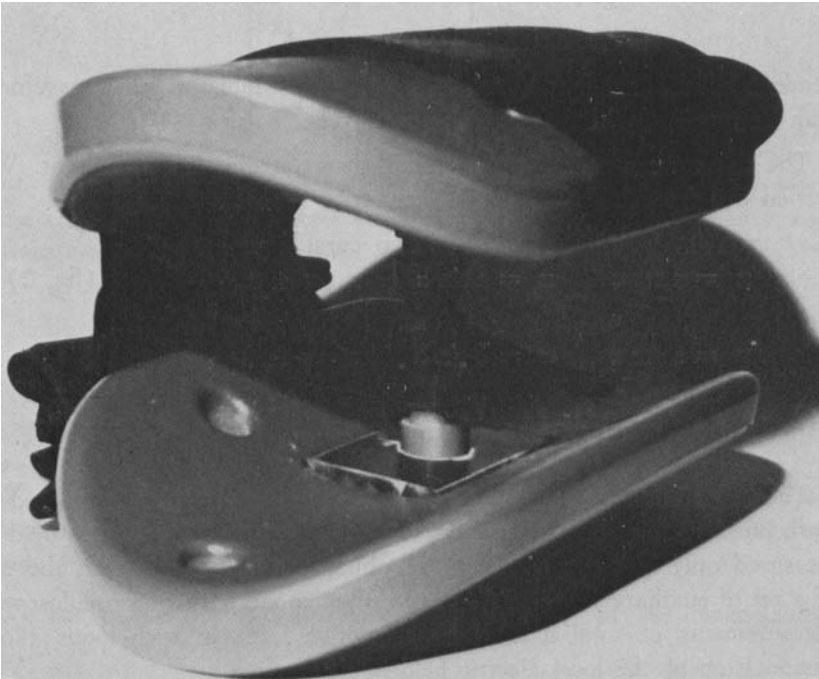


FIG. 3. — Clinical stimulator.

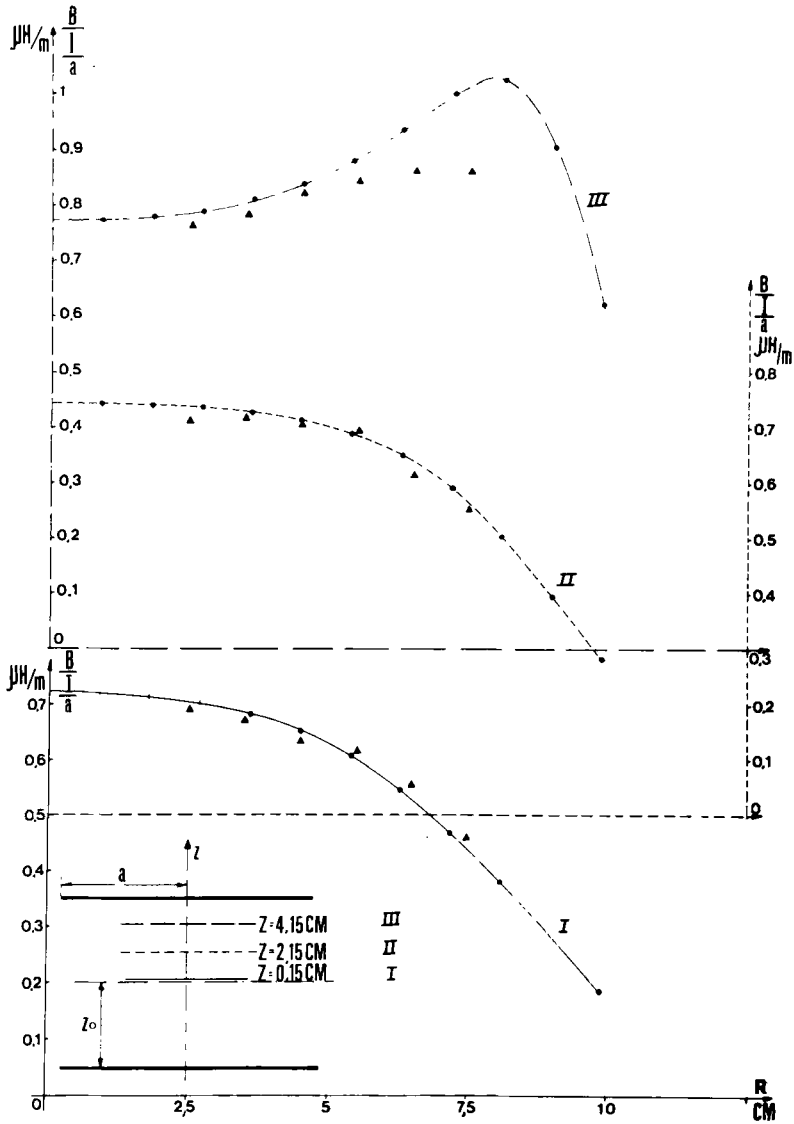


FIG. 4. — Induction field $\frac{B}{I} \frac{1}{a}$ for the stimulator composed of two circular coils in function of radius (R) and in three horizontal planes.

(I) ——— $z = 0.15$ cm (II) - - - - $z = 2.15$ cm (III) - - - - $z = 4.15$ cm
 ($z_0 = 6$ cm ; $a = 9$ cm). $\blacktriangle \blacktriangle$ = experimental values

IV. Results

1. Circular coils.

The topography of the induction field is studied in planes perpendicular to the z axis of the cylinder delimited by the two coils.

The radius a of the coils is 9 cm long and the distance between the coils along the z axis is 12 cm.

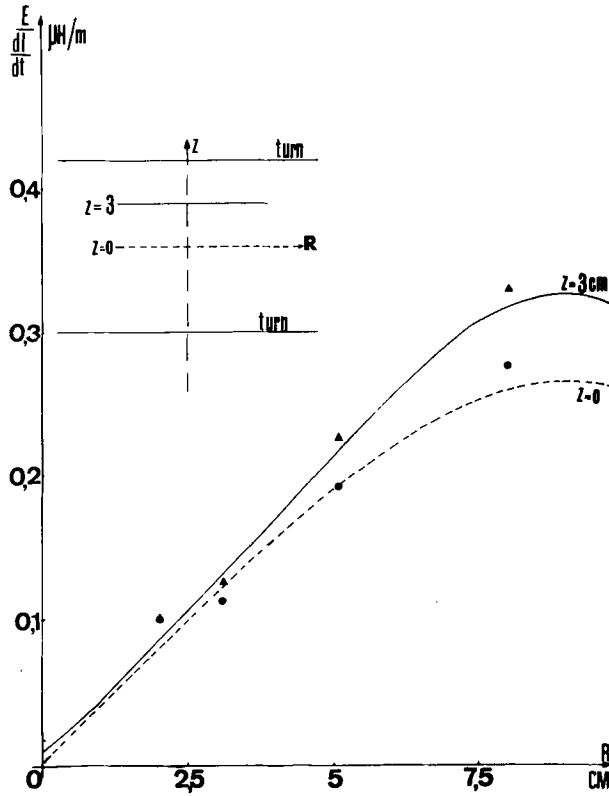


FIG. 5. — Electric field $\frac{E}{dI}$ for the stimulator composed two circular coils as a function of radius and in two horizontal planes.
 (I) ——— $z = 3$ cm
 (II) - - - - $z = 0$
 ($z_0 = 6$ cm ; $a = 9$ cm).
 ▲▲ = experimental values
 ●● = experimental values

The computed and the measured values are roughly the same except in the vicinity of the loops. Figure 4 shows these values in three planes, one close to the middle between the two coils and two others at 2.15 cm and 4.15 cm from the middle.

The induction field $\frac{B}{I}$ is homogenous around the z axis.

$$\frac{B}{I} = \frac{1}{a}$$

The electrical field $\frac{E}{\frac{dI}{dt}}$ is also studied in the same coil configura-

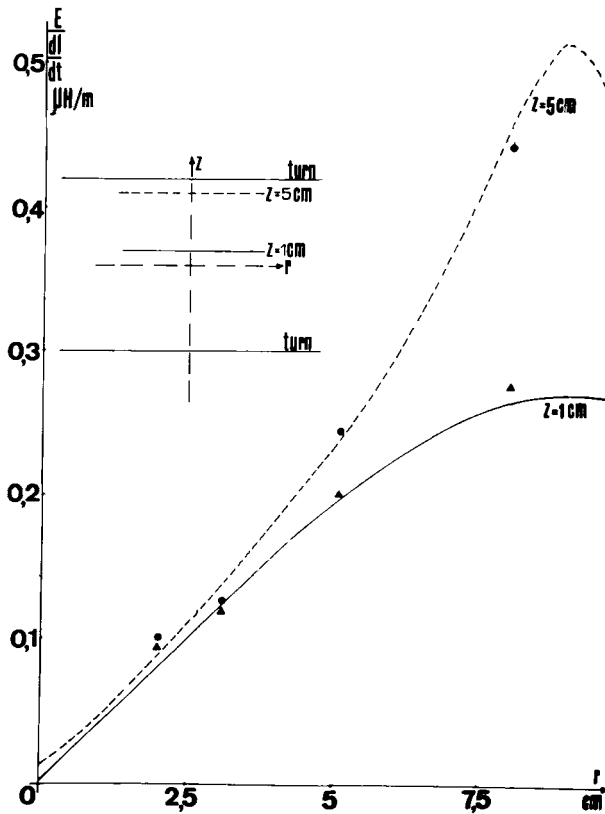


FIG. 6. — Electric field for the stimulator composed of two circular turns as a function of radius and in two horizontal planes.

(I) ——— $z = 1$ cm

(II) - - - - $z = 5$ cm

($z_0 = 6$ cm ; $a = 9$ cm).

▲▲ = experimental values

●● = experimental values

tion. Figures 5 and 6 show these values for planes in the middle ($z = 0$) and at 1 cm, 3 cm and 5 cm from the middle. The amplitude of the electrical field is close to zero around the z axis. The highest values are facing the loops of the coils.

2. Clinical coils.

The topography of the induction field is studied in a system of rectangular coordinates x, y, z with the origin on the central point of the coils (see figures 7 to 11).

Figure 7 gives the computed and the measured values for different parallels to the y axis in the plane $x = 0$ and figure 8 in the plane $x = 3.133$ cm. On both figures the diagrams on the right show the cross section of the coils with the values of the principal dimensions.

The diagrams on figures 9, 10 and 11 show the lateral aspect of the coils. The amplitude of the induction field is given for parallels to the x axis and in the plane $z = 0$ (in the middle between the two coils), $z = 2.7$ cm and $z = 5.4$ cm.

Figures 12, 13 and 14 show the amplitude of the electrical field on parallels to the x axis in planes characterized by different values of z .

To summarize these data we consider a schematic representation of the clinical coils (fig. 15).

The coils define between them a parallelepipedic volume where different planes parallel to the symmetry plane of the coils are studied.

The plane n° 1 is the plane $z = 0$ (fig. 16).

For each of the six planes the profil of the amplitude of the induction field (fig. 17) and of the electrical field (fig. 18) are represented in a three dimensional diagram.

The induction field is roughly homogenous and higher in the central area of the plane $z = 0$. Amplitude spikes appear in the vicinity of the wires of the coils.

On the contrary the electrical field is close to zero in the central area of the plane $z = 0$ and then increase to reach a maximum facing the edges of the coils.

The same spikes are observed in the vicinity of the wires of the coils. This area is the most heterogenous for the induction field as well as for the electrical field.

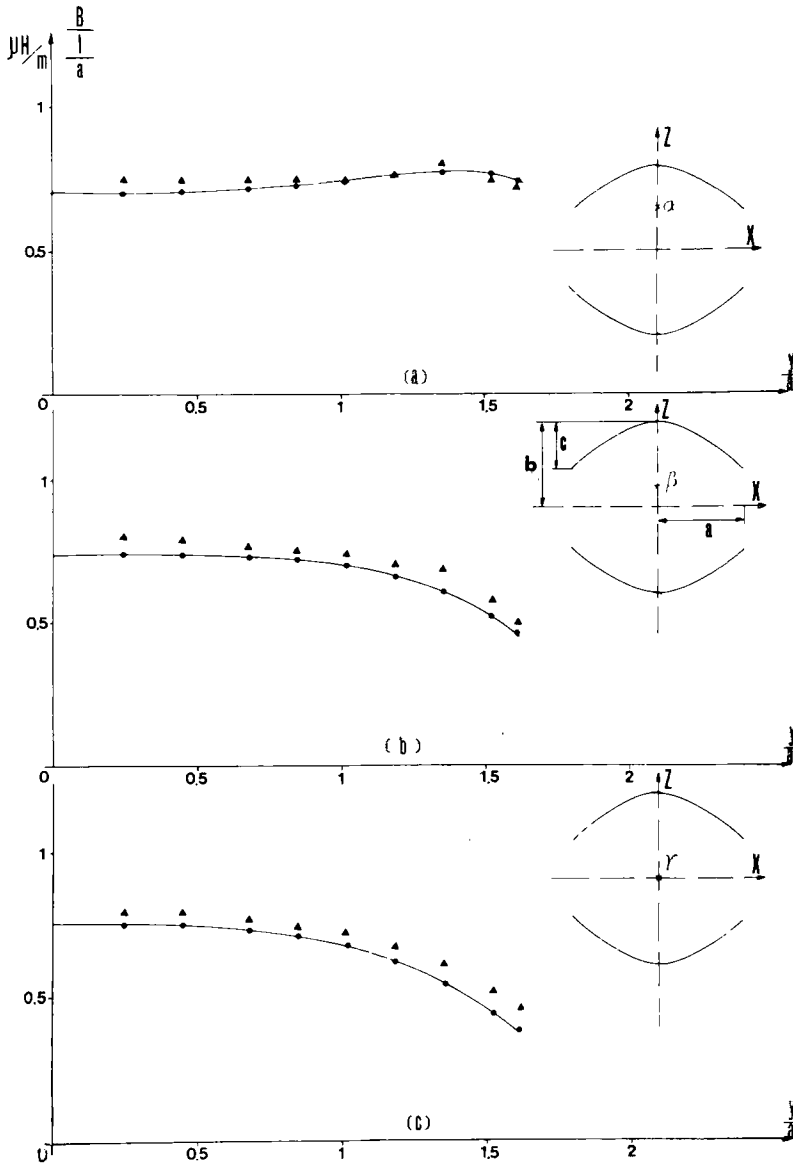


FIG. 7. — Induction field for the clinical stimulator as a function of the cartesian coordinate y in the plane $x = 0$ for 3 different parallels to the y -axis defined by α) $z = 3$ cm β) $z = 1.5$ cm γ) $z = 0$

▲ experimental values

($a = 5.9$ cm $b = 10.9$ cm $c = 3.15$ cm $z_0 = 5.9$ cm)

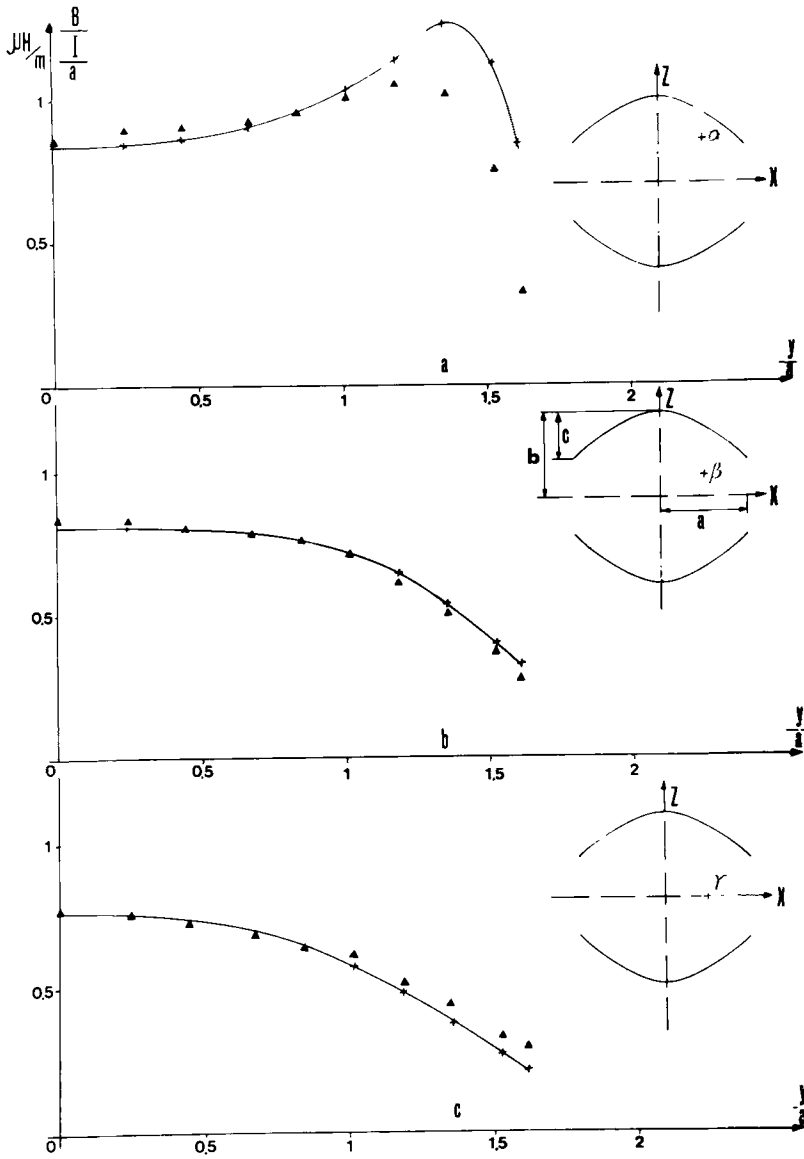


FIG. 8. — Induction field for the clinical stimulator as a function of the cartesian coordinate y in the plane $x = 3.133$ cm for 3 different parallels to the y -axis defined by

- α) $z = 3$ cm β) $z = 1.5$ cm γ) $z = 0$
- ▲ experimental values

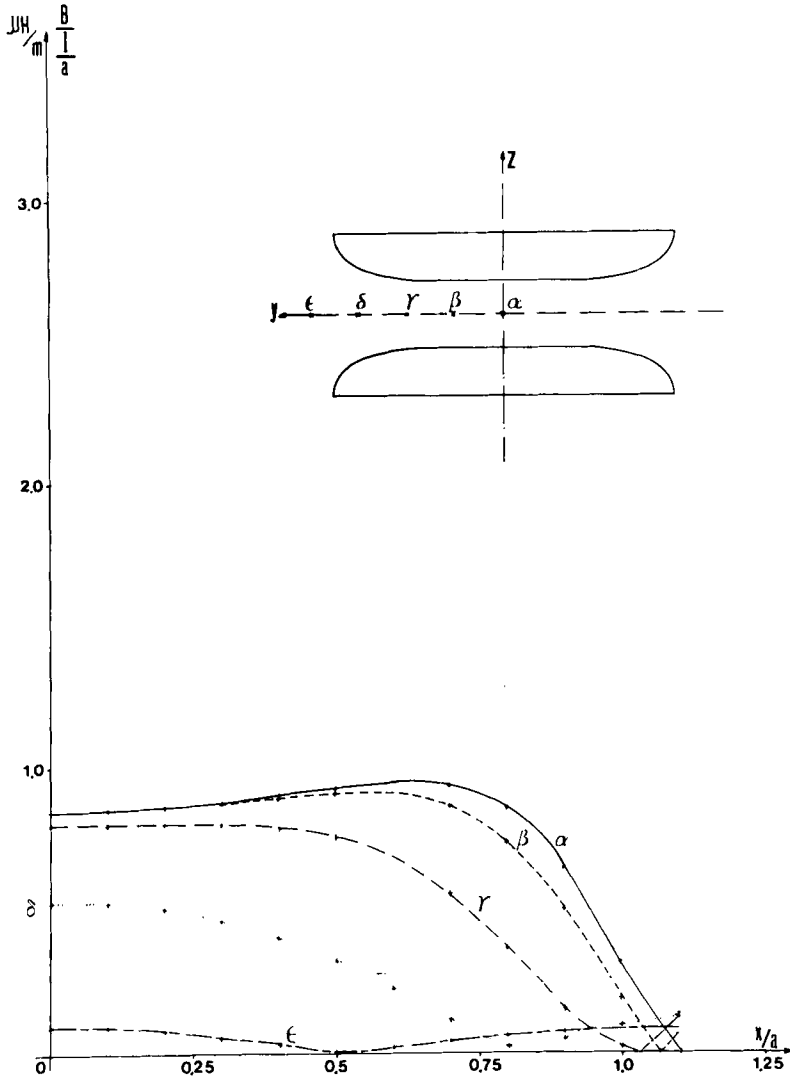


FIG. 9. — Induction field for the clinical stimulator as a function of cartesian coordinate x in the plane $z = 0$ for 5 different parallels to the x -axis defined by

α) ————— $y = 0$ δ) $y = 8.85$ cm
 β) - - - - - $y = 2.95$ cm ϵ) - . . . - $y = 11.8$ cm
 γ) - . - . - $y = 5.9$ cm

($a = 6.75$ cm $b = 12$ cm $c = 3.5$ cm $z_0 = 5.75$ cm).

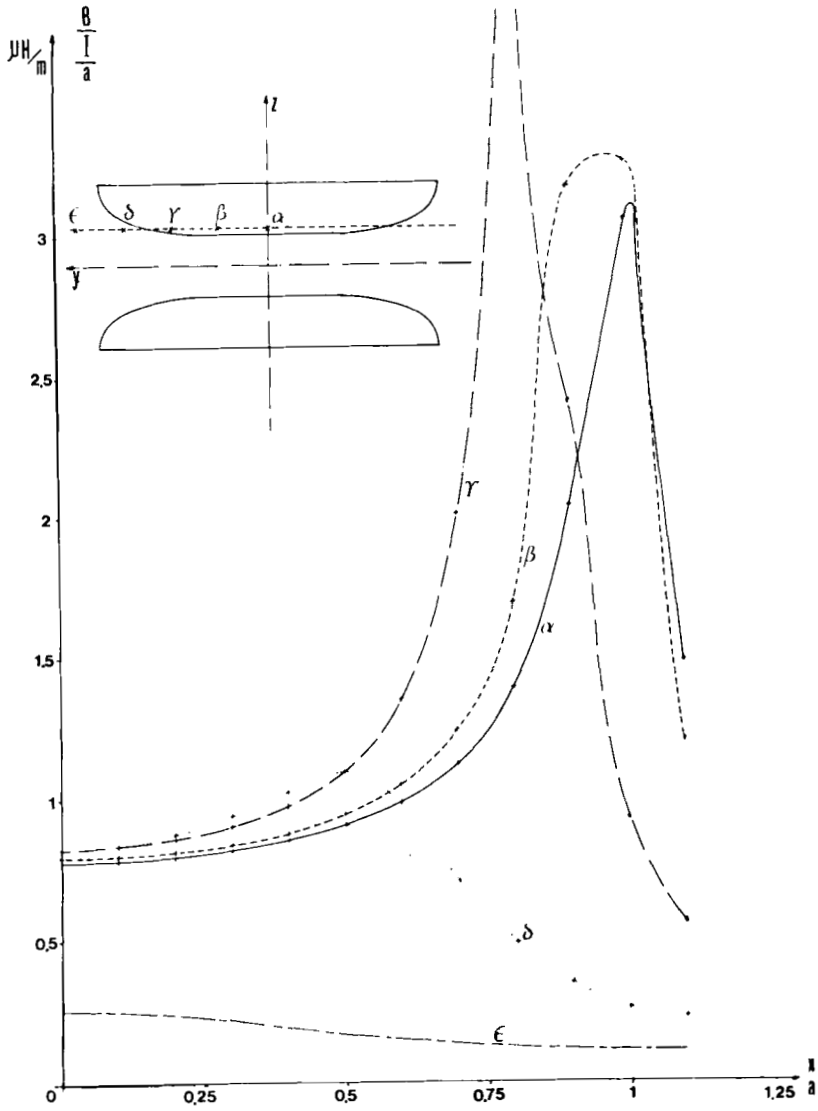


FIG. 10. — As figure 9 but plane $z = 2.70$ cm.

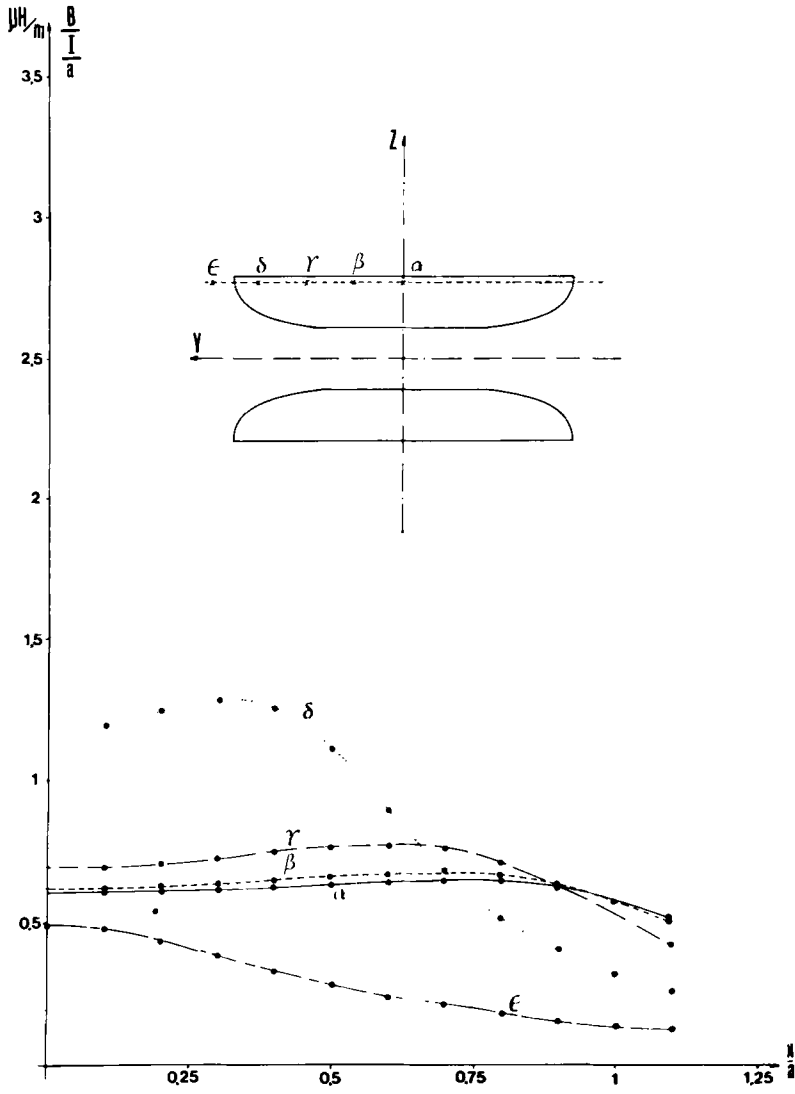


FIG. 11. — As figure 9 but plane $z = 5.4$ cm.

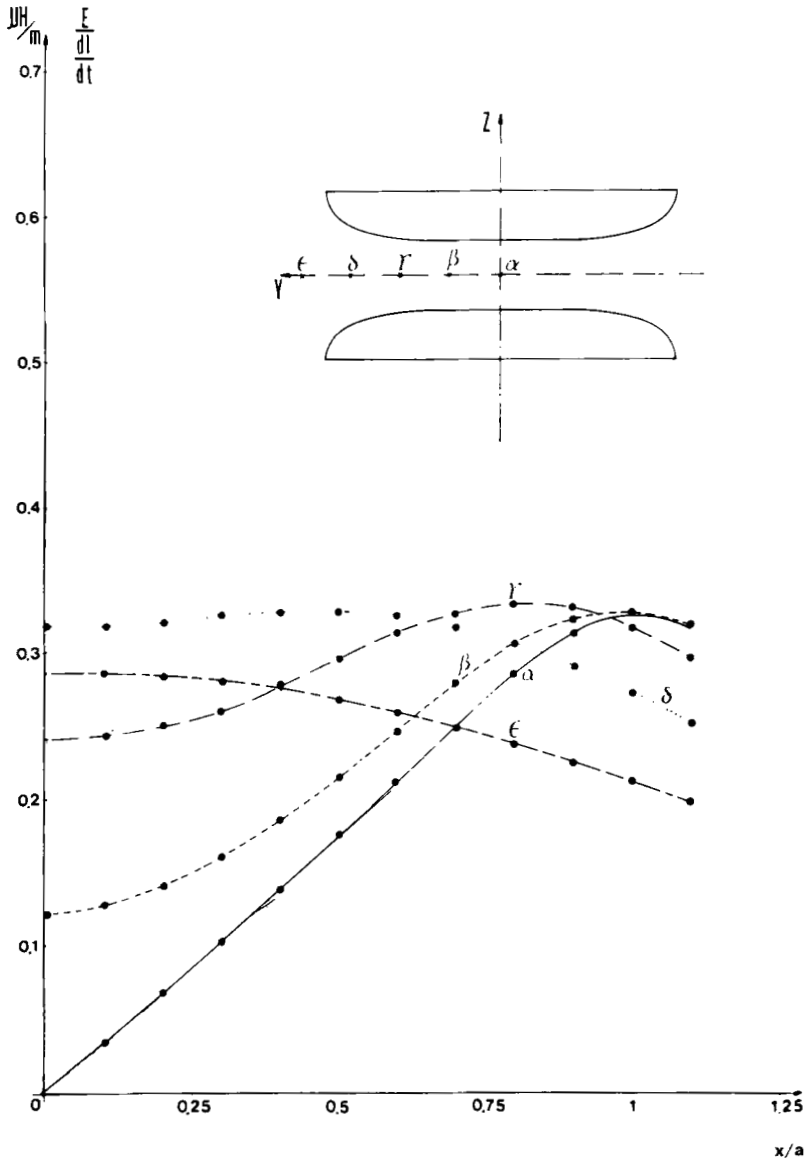


FIG. 12. — Electric field for the clinical stimulator as a function of cartesian co-ordinate x in the plane $z = 0$ for 5 different parallels to the x -axis defined by :
 α) ————— $y = 0$ γ) - . - . - $y = 5.9$ cm ϵ) - - - - $y = 11.8$ cm
 β) - - - - - $y = 2.95$ cm δ) $y = 8.85$ cm
 ($a = 6.75$ cm $b = 12$ cm $c = 3.5$ cm $z_0 = 5.75$ cm).

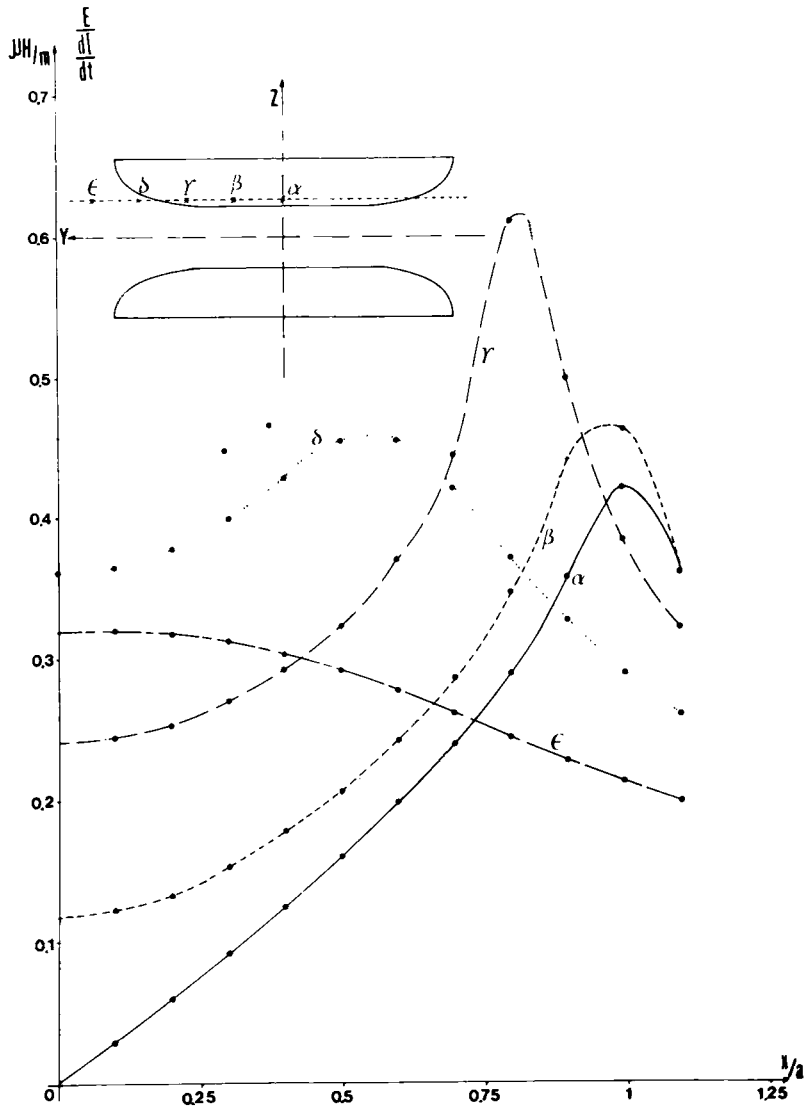


FIG. 13. — As figure 12 but plane $z = 2.7$ cm.

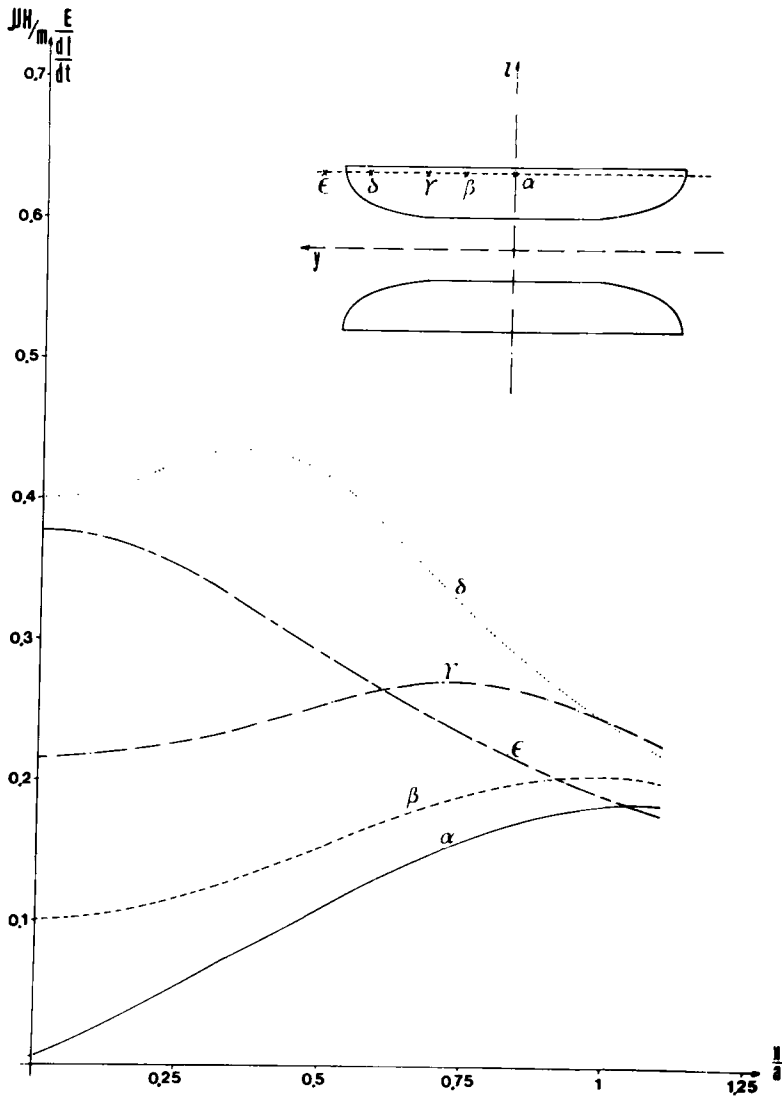


FIG. 14. — As figure 12 but plane $z = 5.4$ cm.

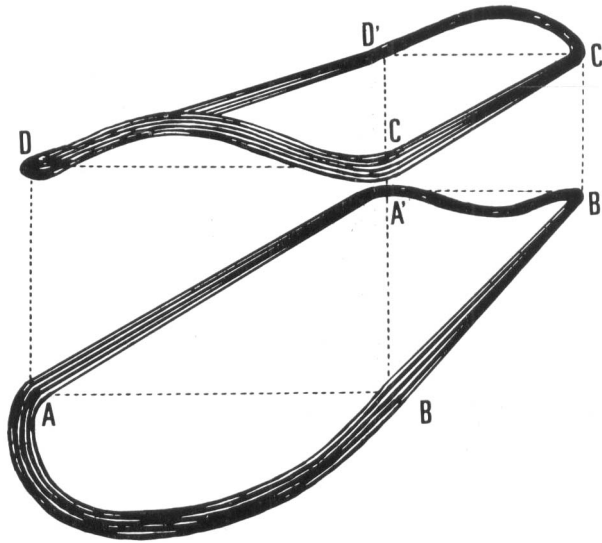


FIG. 15. — View of stimulator with an enveloped parallelepipedic box in dotted lines.

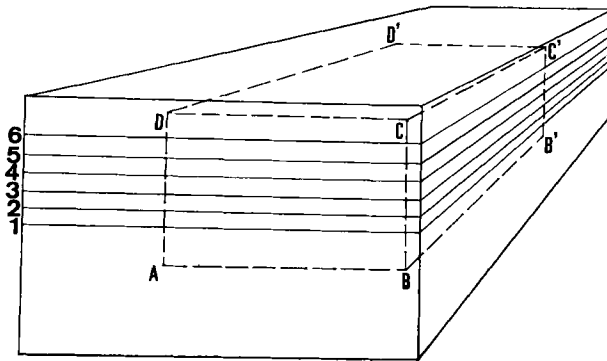


FIG. 16. — The same box in dotted lines and six planes where the values of the field strengths are defined.

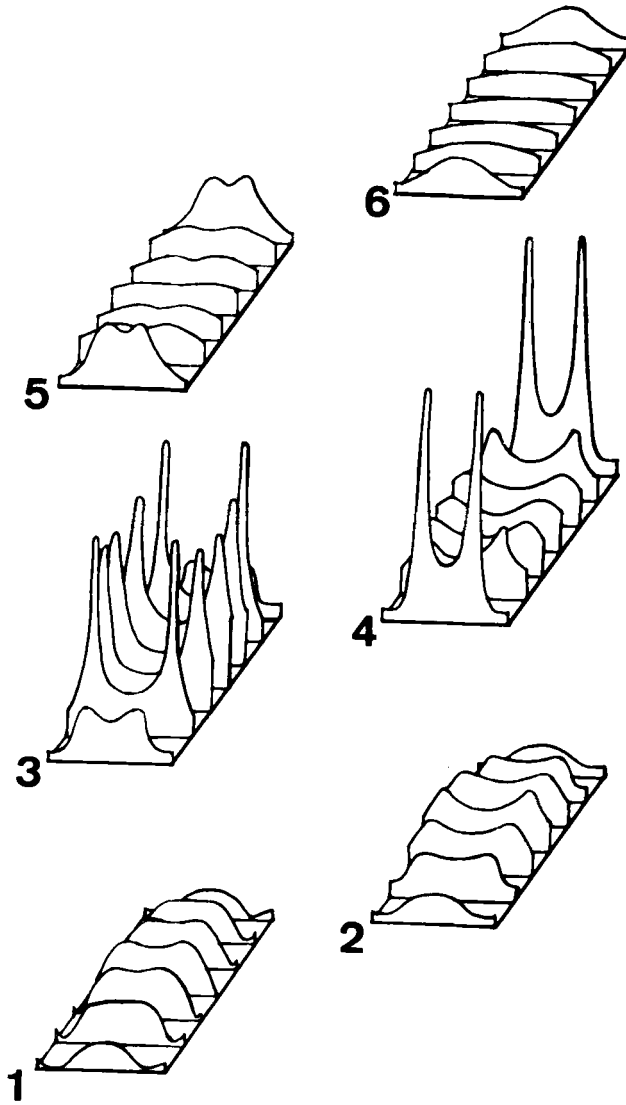


FIG. 17. — Perspective view of induction field in six planes defined by figure 16.

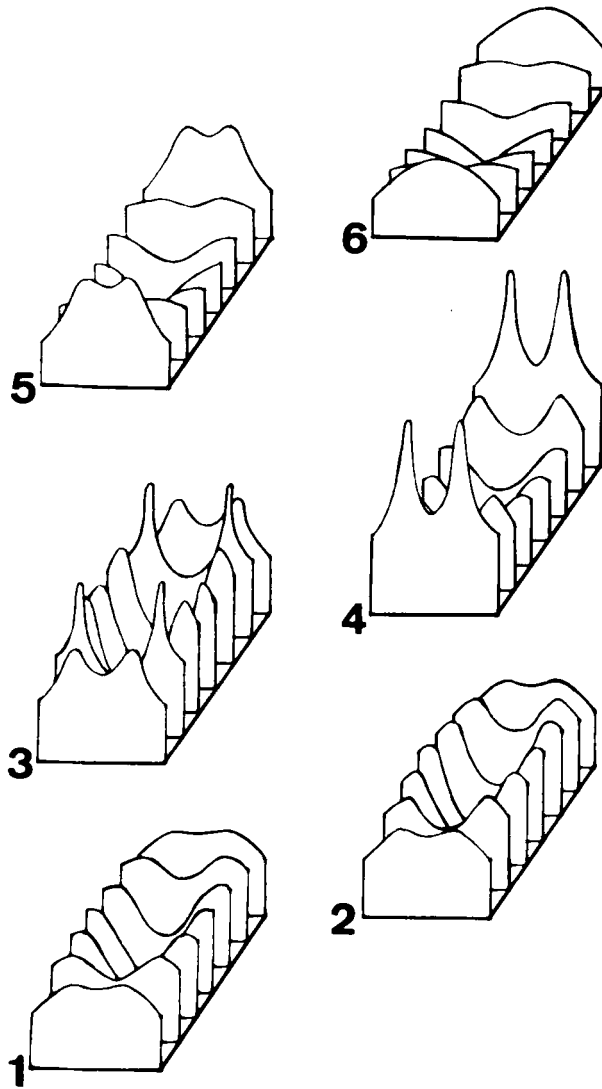


FIG. 18. — Perspective view of electric field in the six planes defined by figure 17.

V. Conclusions

It is possible to compute numerically the induction field and the electric field in the space determined by a stimulator composed of two coils of any geometrical shape. Results make obvious that the induction field is almost uniform in the central part of the stimulator. On the contrary, the electric field is almost zero in the central part of the stimulator and increases very rapidly to the outlines of the apparatus and then begins to decrease.

From these results, it appears necessary to know the active parameter in the electromagnetic stimulation.

The position of the coils allowing the best stimulation in the pathologic area is different if it is the induction field or the electrical field which is acting on the cell activity.

It is fundamental to realize these computations and measurements for each stimulating device before any application.

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