APPENDIX

ITERATIVE PROCEDURE FOR X-RAY SOURCE LOCALIZATION

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A description of the socalled iterative procedure for the locating of the source has to be preceded by a detailed description of the proportions of elements of the internal construction of the cage. Thus, insight may be gained into the various corrections that have to be applied to the X-ray coordinates in order that the X-ray coordinates and the spatial coordinates may be considered in relation to the same system of XYZ coordinates. As described in Chapter 4.3, the cage contains three edges of which constitute a reference system XYZ with origin O_s (see Figure A-1). These edges are situated at a predetermined small distance from the various X-ray planes. In this experiment, the X-ray tubes were mounted in such a way that the edges were projected obliquely on the X-ray films. One of the cornerstones of the present investigation is optimal accuracy; therefore, the deviation from the orthogonal projection of these edges on the X-ray planes was calculated. To render this calculation possible, small metal marking strips were attached to the brass rods, and the corners of these strips were used as reference points.
The procedure applied will now be explained with reference to the projections on the vertical plane (see Figure A-1).

List of symbols for cage (see Figure A-1)

X	: edge of brass rod in X direction, topside; farthest away from X-ray plane V (vertical cassette plane)
X'	: edge of brass rod in X direction, topside; closest to X-ray plane V
X''	: edge brass rod in X direction, underside; farthest away from X-ray plane V
X'''	: *vertically projected edge* X'' *on horizontal cassette plane*
Y	: edge brass rod in Y direction, topside; front
Y''	: edge brass rod in Y direction, underside; front
Y'''	: vertically projected edge Y'' on horizontal cassette plane

Z : edge brass rod in Z direction, front; farthest away from X-ray plane V

Z' : edge brass rod in Z direction, front; closest to X-ray plane V

O_s : point of intersection edges X, Y and Z (origin spatial coordinate system)

0' : point of intersection X' and Z' edges (reference point)

0'' : point of intersection X'' and Y'' edges

0''' : point of intersection of Z edge with horizontal cassette plane

0_{HF} : point of intersection X''' and Y''' edges

X_V : point at end of the X' edge (reference point)

L_V : point of intersection of edge of vertical metal marking strip and X' edge (reference point)

S_V : point at end of the Z' edge (reference point)

XLVC : distance L_V to O'

ZSVC : distance S_V to O'

ALVF : distance L_V (or X' edge) to vertical X-ray plane

ZHF : distance Y edge to horizontal X-ray plane

YVF : distance X edge to vertical X-ray plane

X_H : point at end of the X'' edge (reference point)

L_H : point of intersection of edge of horizontal marking strip H_1 and X''' (reference point)

S_H : point of intersection of edge of horizontal marking strip H_2 and Y''' (reference point)

XLHC : distance L_H to O'''

YSHC : distance S_H to O'''

In the cage, the following four reference points have been placed, which are projected on the vertical plane V by source B_1:

1. reference point X_V; this is the end of edge X';
2. reference point L_V; this is the common point, closest to the origin O' of the metal marking strip and the edge X';
3. reference point O'; this is the point of intersection of edge X' and edge Z';
4. reference point S_V; this is the end of edge Z'.

Source B_1 projects these reference points in the vertical X-ray film in XVF, LVF, OVF and SVF, respectively. The deviation of the real projection with respect to the orthogonal projection of the reference points might be computed if the positions of the X-ray sources were known.

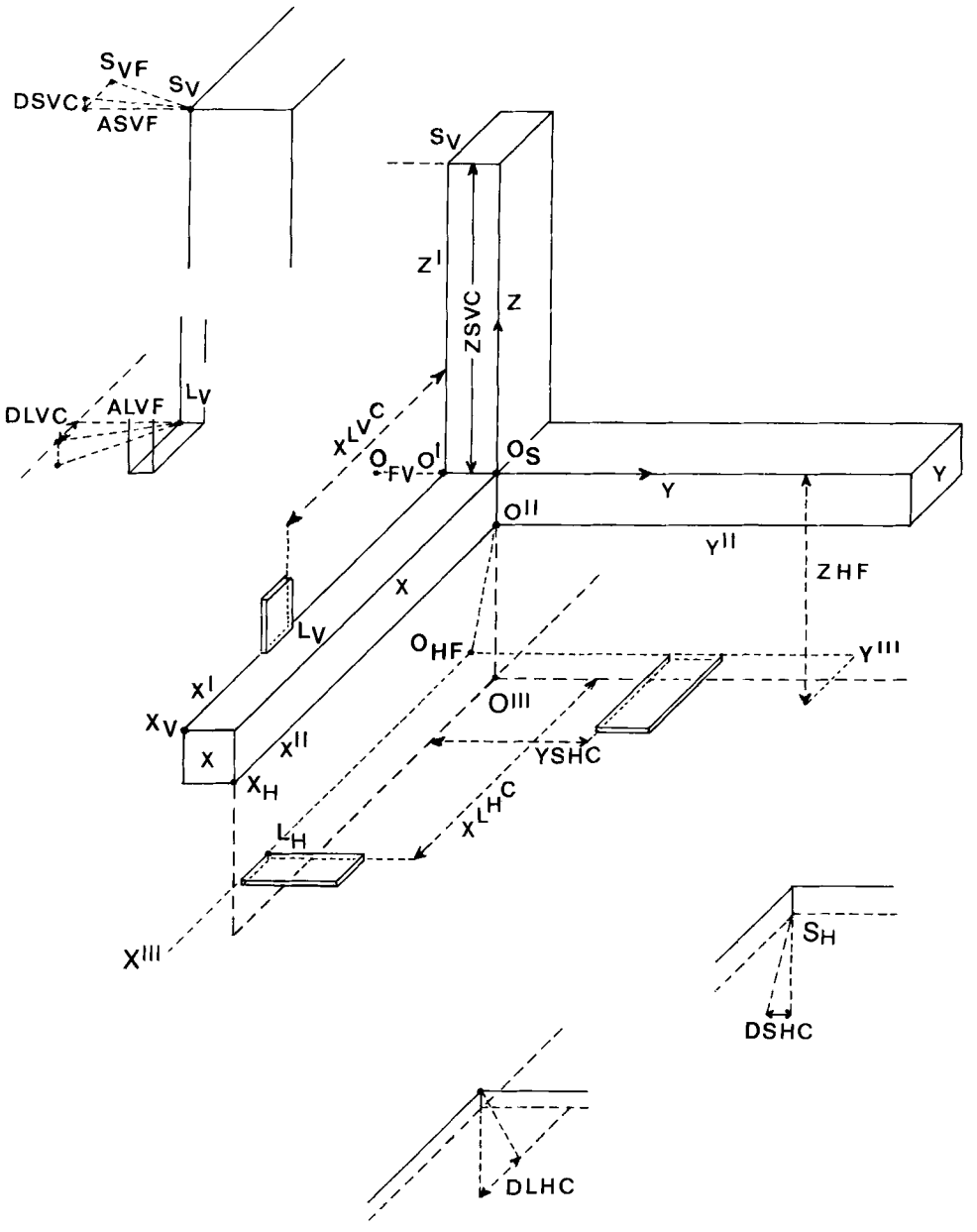

Figure A-1. Spatial relations experimental cage.

Since it is just the position of the source that we want to calculate, we apply the following procedure:

a. estimate the deviation of reference point L_V (in the positive X direction). This deviation we call DLVC (deviation L_V coordinate).
b. estimate the deviation of reference point S_V (in the positive Z direction). This deviation we call DSVC (deviation S_V coordinate).
   These estimates may be made fairly accurately, because the reference points are situated close to the X-ray plane V.
c. on the basis of the estimates of DLVC and DSVC, compute the coordinates of the four reference points with respect to O_S, after which the position of source B_1 can be computed from these four points and the known coordinates of the balls P_2 and P_3 (source locating, first time).
d. on the basis of the coordinates of source B_1, just computed, compute the deviation of L and S. This gives us a new DLVC and DSVC; these will probably not have the same values as the estimated ones!
e. subsequently, on the basis of this new DLVC and new DSVC and of the coordinates of the balls P_2 and P_3, once again compute the position of source B_1 (source locating, second time).

This second computation of the position of source B_1 will give a smaller error than the first computation. Why is it that repetition of the source location gives a more accurate result? Figure A-2 shows the situation viewed from above. It indicates the actual position of source B_1 and its incorrectly estimated position as used for the first source-locating, at a distance ΔX_1, from B_1. The ratio of ΔX_1 to the error in DLVC is the ratio of the distance between L_V and B_1 to the distance between L_V and the X-ray plane V (ALVF). The error in DLVC affects the locating of the projections of P_2 and P_3 in the X-ray film. For the source-locating, second time, it may be stated that the ratio of the error in the X-coordinate of the source (ΔX_2) and the error in DLVC equals the ratio of the distance between P_3 and source B_1 to the distance between P_3 and X-ray plane V.

Figure A-2. Relation of source position B_1 and its inaccurately estimated positions with errors ΔX_1 and ΔX_2, and distances FP_3 and DY.

ΔX_1	: error in the firstly calculated position B_1 based on estimate of projection L_V
ΔX_2	: error in the secondly calculated position of B_1 based on the partly corrected estimate of projection L_V
DY	: distance L_V to B_1
ALVF	: distance L_V to V
FP_3	: distance P_3 to V

In equation form:

$$\frac{\Delta X_1}{\text{error DLVC}} = \frac{\text{distance } L_V - B_1}{\text{distance } L_V - V} = \frac{DY}{ALVF}$$

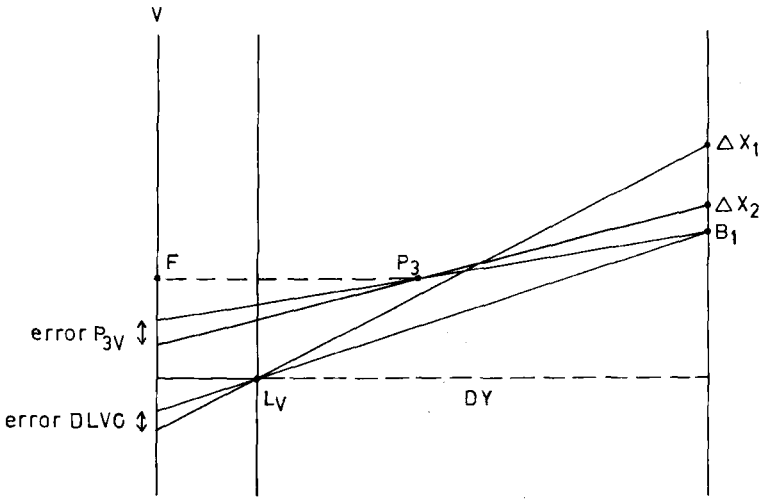

Figure A-2. Relation focus position B_1 and the estimated positions ΔX_1, ΔX_2 and distances FP_3 and DY.

$$\text{error in DLVC} = \Delta X_1 \frac{ALVF}{DY}$$

$$\frac{\Delta X_2}{\text{error in localization projection } P_3} = \frac{\text{distance } P_3 - B_1}{\text{distance } P_3 - V}$$

Since the error in the localization of projection P_3 equals the error in DLVC, the following equations apply:

$$\Delta X_2 = \frac{B_1P_3}{FP_3} \bullet \text{error in DLVC}$$

$$\Delta X_2 = \frac{B_1P_3}{FP_3} \bullet \Delta X_1 \bullet \frac{ALVF}{DY}$$

$$\Delta X_2 = \Delta X_1 \bullet \frac{B_1F_3}{FP_3} \bullet \frac{ALVF}{DY}$$

The following distances, determined by measuring tape and vernier callipers are known:

B_1P_3 = approx 125 cm
FP_3 = approx 15 cm
ALVF = approx 0.14 cm
DY = approx 140 cm

$$\Delta X_2 = \frac{125}{15} \bullet \frac{0.14}{140} = \text{approx} \quad \frac{1}{125} \bullet \Delta X_1$$

Accordingly, the error ΔX_2 depends largerly on the ratio $\frac{ALVF}{FP_3}$.

In other words: since L is situated much more closely to X-ray plane V than the balls of the source-locating apparatus, the error in the second source locating will be smaller than that in the first locating. This mechanism continues to make itself felt: a third source locating gives an even more accurate position than the second, etc.
In our procedure, we have computed the X-coordinate of the source four times. Since the difference between the X coordinates as determined the fourth time and the third time did not exceed 0.01 mm, we have chosen the third-time X coordinate for the subsequent computation procedure.

In X-ray plane V, deviation occurred in the X as well as in the Z direction. Clearly, the reasoning that applies to deviation in the X direction applies equally to the deviation in the Z direction. A survey of the differences in the computed X and Z coordinates of source B_1 as observed during repeated source locating is presented in Table A-1.

The relations between the coordinates of the reference points, source B_1 and the projections of the reference points on plane V are explained in Figure A-3.
Figure A-3 shows a view from above. Reference points O_V and L_V are projected by B_1 on the vertical X-ray plane V.
The drawing shows:

1. DLVC = deviation in the positive X direction of reference point L_V in mm
2. XLVC = distance between reference point O_V and reference point L_V, measured with vernier callipers in cage, in mm
3. XLVF = the X coordinate of L_V in mm, as measured in the X-ray film (= XLVC + DLVC - COXV)
4. COXV = the correction in mm to be applied to the X-ray coordinates if we want to calculate these X-ray coordinates (in the X direction) in relation to O_S.

List of symbols for X-ray photographs (see Figures A-1 and A-3)

O_{VF} : oblique projection O_V on vertical X-ray plane V from B_1
O_{FV} : perpendicular projection O_V on X-ray plane V
L_{VF} : oblique projection L_V on vertical X-ray plane V from B_1
X_{VF} : oblique projection X_V on vertical X-ray plane V from B_1
S_{VF} : oblique projection S_V on vertical X-ray plane V from B_1
O_{HF} : oblique projection O'' on horizontal X-ray plane H from B_2
L_{HF} : oblique projection L_H on horizontal X-ray plane H from B_2

Table A 1. Coördinates X-ray sources (repeated calculation).

DLVC	DSVC	X B_1	Y B_1	Z B_1	Interlinear distance
0.02	0.00	101.59	1142.64	112.13	0.15
0.07	0.01	101.80	1143.03	112.52	0.17
0.07	0.01	101.80	1143.03	112.52	0.17
0.07	0.01				
DLHC	DSHC	X B_2	Y B_2	Z B_2	Interlinear distance
0.32	0.06	378.30	61.08	1287.16	0.57
0.22	0.03	377.60	60.83	1287.12	0.72
0.22	0.03	377.60	60.82	1287.12	0.72
0.22	0.03				

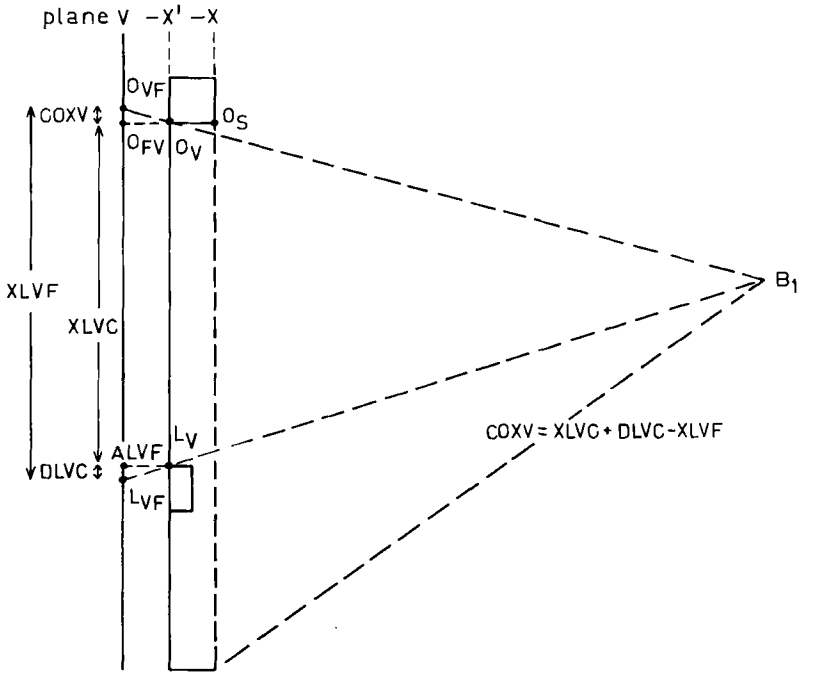

Figure A-3. Relation coördinates and reference points in vertical X-ray photograph.

This correction has a negative sign (deviation in the negative direction of O_V). The figure (Figure A-3) shows:

$$COXV = XLVC + DLVC - XLVF \qquad 1)$$

Analogously, the correction of the deviation in the Z direction follows from:

$$COZV = ZSVC + DSVC - ZSVF \qquad 2)$$

Deviation occurs also in the horizontal X-ray film, but now in the X and Y directions. Owing to the oblique direction of the radiation from source B_2, at an angle of approx 15° with the vertical, these deviations will be larger. For the horizontal plane, reference points L_H and S_H were used. After initial estimation of the deviations of L_H and S_H, source B_2 was located repeatedly as described above for B_1.

Table A-2. Survey of HMTA data combination of upper four balls.

Pin no.	Point no.		0	5	10	15	20	25	30	35
1	P_1	X	73.33	70.39	67.67	65.23	63.03	61.14	59.51	58.20
		Y	24.50	26.99	29.69	32.61	35.76	39.04	42.52	46.09
		Z	79.81	79.79	79.80	79.81	79.79	79.78	79.79	79.77
	P_2	X	57.98	55.18	52.73	50.65	48.96	47.65	46.68	46.18
		Y	25.46	29.30	33.31	37.50	41.92	46.41	51.05	55.70
		Z	87.89	87.87	87.87	87.87	87.88	87.87	87.86	87.84
3	P_3	X	70.79	67.17	63.81	60.73	57.97	55.54	53.38	51.62
		Y	16.68	19.43	22.44	25.71	29.30	33.05	37.05	41.18
		Z	104.16	104.13	104.13	104.13	104.14	104.13	104.12	104.11
	P_4	X	84.98	81.85	78.85	76.03	73.41	70.99	68.74	66.76
		Y	22.81	24.27	25.99	27.96	30.17	32.60	35.26	38.05
		Z	95.82	95.78	95.80	95.80	95.80	95.79	95.79	95.78
Distance	P_1P_2		17.38	17.37	17.37	17.37	17.36	17.37	17.39	17.38
	P_1P_3		25.70	25.68	25.68	25.68	25.69	25.69	25.68	25.69
	P_1P_4		19.87	19.86	19.86	19.85	19.87	19.87	19.84	19.86
	P_2P_3		22.49	22.48	22.49	22.47	22.47	22.48	22.47	22.47
	P_2P_4		28.27	28.27	28.27	28.25	28.26	28.26	28.26	28.25
	P_3P_4		17.56	17.57	17.56	17.57	17.57	17.57	17.57	17.56
Distance of crossing straight lines	P_1		0.11	0.05	0.07	0.10	0.07	0.07	0.09	0.13
	P_2		0.10	0.06	0.08	0.10	0.10	0.09	0.10	0.14
	P_3		0.14	0.05	0.10	0.11	0.09	0.07	0.08	0.12
	P_4		0.14	0.08	0.09	0.11	0.10	0.09	0.09	0.12

Table A-3. Survey of HMTA data combination of lower four balls.

Pin no.	Point no.		0	5	10	15	20	25	30	35
1	P_1	X	46.09	43.44	41.23	39.49	38.20	37.39	37.03	37.15
		Y	26.72	31.60	36.61	41.80	47.18	52.56	58.07	63.54
		Z	43.71	43.69	43.68	43.68	43.67	43.67	43.65	43.64
	P_2	X	53.77	52.48	51.54	51.00	50.84	51.03	51.58	52.50
		Y	42.50	46.64	50.83	55.05	59.36	63.63	67.90	72.07
		Z	44.78	44.77	44.75	44.76	44.75	44.74	44.73	44.72
3	P_3	X	41.58	39.43	37.75	36.55	35.86	35.65	35.89	36.65
		Y	32.29	37.55	42.90	48.37	53.97	59.55	65.19	70.73
		Z	57.70	57.68	57.67	57.65	57.65	57.66	57.64	57.63
	P_4	X	57.24	55.03	53.16	51.65	50.52	49.77	49.37	49.37
		Y	32.05	35.92	39.93	44.06	48.35	52.69	57.12	61.52
		Z	49.57	49.55	49.55	49.55	49.53	49.54	49.53	49.52
Distance	P_1P_2		17.58	17.59	17.59	17.58	17.58	17.59	17.60	17.60
	P_1P_3		15.72	15.72	15.73	15.71	15.71	15.73	15.74	15.74
	P_1P_4		13.68	13.69	13.70	13.69	13.69	13.70	13.70	13.71
	P_2P_3		20.50	20.49	20.49	20.48	20.49	20.50	20.49	20.49
	P_2P_4		12.01	12.01	12.02	12.00	12.00	12.01	12.01	12.01
	P_3P_4		17.67	17.67	17.67	17.66	17.67	17.67	17.68	17.68
Distance of crossing straight lines	P_1		0.03	0.02	0.04	0.05	0.05	0.08	0.07	0.09
	P_2		0.07	0.03	0.05	0.07	0.08	0.08	0.06	0.11
	P_3		0.06	0.03	0.05	0.08	0.09	0.08	0.07	0.13
	P_4		0.05	0.01	0.04	0.05	0.07	0.06	0.07	0.09

Table A-4. Survey of measuring plate data.

Pin no.	Point no.		0	5	10	15	20	25
		X	55.89	74.55	122.72	45.59	38.75	114.85
	P_1	Y	15.66	13.46	61.40	60.36	14.35	13.71
		Z	17.27	91.98	11.69	11.77	34.01	30.67
1								
		X	93.49	89.91	47.75	45.64	113.20	114.51
	P_2	Y	14.15	88.84	43.24	39.73	14.94	90.16
		Z	84.60	86.95	10.82	86.18	13.75	20.18
		X	73.28	53.17	47.14	120.25	114.82	40.44
	P_3	Y	87.93	87.70	59.66	58.68	87.99	88.44
		Z	92.65	19.24	86.03	87.02	37.95	40.73
3								
		X	69.30	71.72	122.49	120.83	39.17	39.32
	P_4	Y	91.28	12.92	42.71	44.21	90.45	15.34
		Z	15.55	15.17	86.32	11.19	23.19	16.33
	P_1P_2		77.13	77.09	77.14	77.21	77.16	77.18
	P_1P_3		105.86	106.12	106.03	106.02	105.94	105.95
	P_1P_4		76.84	76.87	76.94	76.96	76.87	76.90
Distance	P_2P_3		76.92	77.05	76.99	76.98	76.97	76.90
	P_2P_4		106.30	106.05	106.24	106.29	106.17	106.14
	P_3P_4		77.27	77.15	77.24	77.20	77.11	77.08
Distance	P_1		0.09	0.10	0.17	0.22	0.09	0.16
of	P_2		0.07	0.14	0.17	0.15	0.21	0.08
crossing	P_3		0.13	0.22	0.21	0.14	0.03	0.22
straight	P_4		0.13	0.16	0.26	0.36	0.23	0.10
lines								