CHAPTER VII

ERRORS IN COMPUTATION OF HELICAL MOVEMENTS

7.1 HMTA ANALYSIS

An impression of the influence of the errors of the X-ray photogrammetric data on the computed helical movement may be gained from the use of the testing apparatus, the HMTA described in Chapter 6.
The marking balls of the HMTA rotate in steps of approx 5° around a vertical axis with a translation of approx 0 mm. From the positions calculated by X-ray photogrammetry (Chapter 3), the helical axes are now computed. The results of these computations give the angle π (angle axis in base plane), the angle φ (angle axis with base plane), the angle of rotation α and a translation t. The findings are listed in Table 7-1.

Table 7-1. Results calculation helical movements HMTA upper and lower combination, 5° stepsize.

Step	π	φ	α	α_{exp} - om	t
0- 5°	221.50°	89.48°	5.11°	+0.112°	0.02 mm
5-10°	40.45°	89.63°	4.94°	-0.030°	0.00 mm
10-15°	68.76°	89.43°	4.95°	-0.016°	0.02 mm
15-20°	262.72°	88.87°	5.17°	+0.159°	0.06 mm
20-25°	4.34°	88.42°	4.87°	-0.111°	0.06 mm
25-30°	70.47°	88.48°	4.90°	-0.117°	0.07 mm
30-35°	171.86°	89.56°	5.03°	+0.045°	0.00 mm

Step	π	φ	α	α_{exp} - om	t
0- 5°	289.79°	89.19°	5.06°	+0.062°	0.09 mm
5-10°	350.04°	89.53°	4.92°	-0.050°	0.03 mm
10-15°	63.42°	88.92°	5.02°	+0.052°	0.06 mm
15-20°	202.74°	89.29°	5.07°	+0.059°	0.02 mm
20-25°	343.74°	89.17°	4.92°	-0.061°	0.03 mm
25-30°	48.21°	89.60°	5.00°	-0.017°	0.01 mm
30-35°	258.53°	89.56°	4.99°	-0.005°	0.05 mm

For a helical movement of a (non-existent) ideally constructed HMTA, the angle π would range from 0° to 360°, the angle φ would be 90°, the angle of rotation α would be 5° and the translation d would always be 0 mm. We find in the table that the angle π ranged from 4.34° to 262.72° for the upper combination of balls. For the lower combination, π ranged from 48.21° to 350.0°. For angle φ, the value in the upper combination ranged from 88.42° to 89.63°; mean 89.12°, S.D. ± 0.52°. In the lower combination, φ ranged from 88.92° to 89.60°; mean 89.32°, S.D. ± 0.25°. In other words, for the lower combination values for φ were computed that were slightly nearer 90°, with also a slightly smaller variability than for the upper combination. These differences are not significant ($p = 0.47$). The value of φ, in 14 axis computations with step sizes of approx 5°, always differed less than 1° from 90°. It will be found later that the error in the determination of the direction of the axis with approx 5° step size is about 1° (see page 64). For the angle of rotation α, we computed a minimum of 4.87° and a maximum of 5.17°; mean *4.99°*, SD ± 0.11° for the upper combination, for the lower combination we computed an α of minimally 4.92° and maximally 5.07°; mean 5.00°, S.D. ± 0.06°. Table 7-1 also shows the differences between the optically measured rotations and the computed rotations of the upper and lower combinations. The mean value of this difference amounts to 0.085°, S.D. ± 0.05° for the upper combination and to 0.044°, S.D. ± 0.02° for the lower combination. To sum up, there were no significant differences between the rotations computed for the lower and the upper combination of HMTA balls, nor were there significant differences between the computed rotations and the measured rotations of the HMTA balls ($p=1$). For the upper and lower combination together, a mean difference of 0.06°, S.D. 0.04° was computed. In other words, the error in our computation of the angle of rotation of the HMTA was of the order of magnitude of 0.1° for a rotation of approx 5°.
For the translation t, we computed for the upper combination a minimum of 0 mm and a maximum of 0.07 mm; mean 0.03 mm, S.D. ± 0.03 mm. For the lower combination we computed for t a minimum of 0.01 mm and a maximum of 0.09 mm; mean 0.04 mm, S.D. ± 0.03 mm.
Accordingly, for the lower and the upper combination, no significant differences ($p=0.59$) were found. For the two combinations together, we computed a mean value of 0.04 mm, S.D. ± 0.02 mm. In other words, the mean error of the translation of the HMTA was less than 0.05 mm.

Since theoretically, greater accuracy is to be expected with a step size of approx 10°, we have also computed the angles π, φ and α, and the translation d of the HMTA for a step size of approx 10°, together with the differences with the optically measured angles α_{om} - α_{exp}.
A review of mean values and standard deviations of these factors for step sizes of 5° and 10° is given in Table 7-3. As can be seen in Table 7-2, π varied from 36.68° to 281.40° for the upper combination and from 2.37° to 323.78° for the lower combination.

Table 7-2. Results calculation helical movements HMTA upper and lower combination, 10° stepsize.

Step	π	φ	α	α_{exp-om}	t
0-10°	210.76°	89.92°	10.05°	+0.082°	0.02 mm
5-15°	58.23°	89.55°	9.89°	+0.042°	0.02 mm
10-20°	281.40°	89.67°	10.13°	+0.151°	0.04 mm
15-25°	223.80°	89.18°	10.04°	+0.048°	0.12 mm
20-30°	36.68°	88.74°	9.77°	-0.228°	0.01 mm
25-35°	85.12°	89.28°	9.93°	-0.072°	0.07 mm
Step	π	φ	α	α_{exp-om}	t
0-10°	309.92°	89.45°	9.98°	+0.012°	0.12 mm
5-15°	44.93°	89.36°	9.94°	-0.002°	0.03 mm
10-20°	101.23°	89.67°	10.09°	+0.111°	0.04 mm
15-25°	284.09°	89.77°	9.99°	-0.002°	0.05 mm
20-30°	2.37°	89.48°	9.92°	-0.078°	0.02 mm
25-35°	323.78°	89.87°	9.99°	-0.012°	0.04 mm

For angle φ, the value for the upper combination varied from 88.74° to 89.92°; mean 89.39°, S.D. ± 0.41°. For the lower combination, φ ranged from 89.36° to 89.87°; mean φ 89.60°, S.D. ± 0.20°. In other words, no significant differences (p=0.77) for φ were found, here, either, for the upper and the lower combination. At 12 axis determinations, the difference from 90° was less than 1°. To conclude, as was to be expected, the accuracy of the determination of φ was slightly better, and the variations slightly greater with a step size of approx 10° compared with a step size of approx 5°, see also Table 7-1.

For the angle of rotation α, the minimum was 9.77° and the maximum 10.13°; mean α 9.97°, S.D. ± 0.13° for the upper combination; for the lower combination these figures were minimum 9.92°, maximum 10.09°, mean α 9.99°, S.D. ± 0.06°. Tables 7-2 and 7-3 show the differences between the optically measured rotations and the computed rotations. The mean value of the difference amounts to 0.10°, S.D. ± 0.07° for the upper combination and to 0.036°, S.D. ± 0.05° for the lower combination.
To conclude, no significant differences (p=0.99) between the upper and the lower combination were found for the rotation α.
We conclude that according to Table 7-3, in this series of HMTA experiments, a step size of approx 10° does not give a better accuracy in the computation of the angle of rotation α and the variability than a step size of approx 5°.
For the translation t, we computed a minimum of 0.01 mm and a maximum of 0.12 mm; mean 0.05 mm, S.D. ± 0.04 mm for the upper combination. For the lower combination, these values were 0.02 mm, 0.12 mm, 0.05 mm and ± 0.04 mm, respectively. If we compare the computed translations t for step sizes of approx

Table 7-3. Results calculations mean values and standard-deviations for inclination angle $\overline{\varphi}$, rotation angle $\overline{\alpha}$, standarddeviation SDα, difference-angle $\overline{\alpha}_{exp-om}$, standarddeviation SD$\alpha_{exp-om}$, translation t and standard-deviation SD t.

	$\overline{\varphi}$	SDφ	$\overline{\alpha}$	SDα	$\alpha_{exp-om.}$	SD$\alpha_{exp-om.}$	$\overline{t}$	SD t
Stepsize 5° upper combination	89.12°	0.52°	4.99°	0.11°	0.085°	0.05°	0.03 mm	0.03 mm
Stepsize 5° lower combination	89.32°	0.25°	5.00°	0.06°	0.044°	0.02°	0.04 mm	0.03 mm
Stepsize 10° upper combination	89.39°	0.41°	9.97°	0.13°	0.104°	0.07°	0.05 mm	0.04 mm
Stepsize 10° lower combination	89.60°	0.20°	9.99°	0.06°	0.036°	0.05°	0.05 mm	0.04 mm

5° and approx 10°, we find virtually identical translations of step size 10°, with virtually identical variability. The differences are statistically not significant ($p=0.39$).
To conclude, there is definitely not a decrease of the translation with a larger angle of rotation. A possible explanation is that in the HMTA a real translation is indeed present.

7.2 ANALYSIS OF INTRODUCED ERRORS

This paragraph was written in cooperation with C.W. Spoor.
A second method that we have applied is the estimation of errors by means of a computation model to determine how the accuracy of the computed direction of the helical axis is affected by:
1) minor accidental errors in the determination of the spatial coordinates and
2) the measure of the angle of rotation α.
Results of the computation with this model are not influenced by systemic errors caused by deviations from the norm in dimensions and construction of the testing apparatus. The computation model is a tetrahedron with a side of 100 mm and four angle points that were used as measuring points. This tetrahedron was placed in three different initial positions A, B and C and then rotated around the Z-axis with a known nominal rotation angle, in a number of steps of equal size.

Table 7-4. Coördinates balls test tetrahedron.

Ball	Tetrahedron A			Tetrahedron B			Tetrahedron C		
	X	Y	Z	X	Y	Z	X	Y	Z
1	50	0	0	50	$\frac{-50}{3}\sqrt{3}$	0	$25.\sqrt{2}$	50	0
2	-50	0	0	-50	$\frac{-50}{3}\sqrt{3}$	0	$25.\sqrt{2}$	-50	0
3	0	50	$50\sqrt{2}$	0	$\frac{100}{3}\sqrt{3}$	0	$-25.\sqrt{2}$	0	50
4	0	-50	$50\sqrt{2}$	0	0	$\frac{100}{3}\sqrt{6}$	$-25.\sqrt{2}$	0	-50

Table 7-5. Nominal rotation angles α_{nom}.

	$\alpha_{nom.}$
1	0.1°
2	0.3°
3	1.0°
4	3.0°
5	10.0°
6	30.0°

Table 7-4 lists the initial coordinates of the tetrahedron and Table 7-5 shows the nominal rotation angles used.
With the exception of the initial position, scattering with an error of 0.1 mm was allowed for each of the subsequent steps. Consequently, each of the 12 coordinates had a possibility of 1/3 of an error of +0.1 mm, 0 mm or -0.1 mm along one of the three coordinates. Subsequently, for each step (transition) the helical axis was computed on the basis of the coordinates deviating because of the scattered error. Proceeding from these results, it was attempted to find the relationship between the nominal rotation axis of the step size (α_{nom}) and the direction of deviation of the helical axis computed (δ).
From the three different initial positions, the tetrahedron was rotated to 11 positions in succession. Each series comprised 11 changes of position and consequently, 11 helical axes. Of these 11 helical axes, the mean and the maximal direction of deviation were always determined. This was carried out with each of the six nominal rotation angles. In all, for each nominal rotation

angle and from three initial positions, a scatter was introduced 10 times: 4 times for initial position A, 4 times for initial position B and twice for initial position C. Since 6 different angles were used, a total of 60 series had to be computed for an estimate of the relationship between the measure of the rotation angle and the deviation of the rotation axis.

Table 7-6. Maximal errors of the directions of the helical axes (δ max.) and mean values of these maximal errors ($\bar{\delta}$ max.) for series A, B and C separately for 6 different nominal rotation angles.

δ max.										$\bar{\delta}$		
A				B				C		A	B	C
1	2	3	4	1	2	3	4	1	2	-	-	-
71	-	-	-	-	-	126	-	-	-	-	-	-
44	43	46	35	55	42	49	38	31	43	42	46	37
8.7	10.73	9.47	12.45	9.98	10.43	14.17	11.15	11.83	11.38	10.34	11.43	11.61
3.9	3.53	4.13	4.0	3.97	2.92	3.57	3.83	4.68	4.53	3.89	3.57	4.61
1.37	1.53	1.08	1.08	1.27	1.27	1.1	1.2	1.37	1.07	1.27	1.21	1.22
0.4	0.35	0.28	0.63	0.3	0.42	0.42	0.47	0.37	0.35	0.42	0.40	0.36

A review of the maximal deviations for each of the 10 scatter series is shown in Table 7-6. It is interesting to note that 8 times, at the nominal value of 0.1°, an indeterminate maximal deviation angle δ_{max} was found.

Table 7-7 for each nominal rotation angle lists the highest deviations found of the 10 maxima in Table 7-6. The table shows that the axis deviation δ clearly is inversely proportional to the measure of the nominal rotation angle α: the wider the rotation angle α, the less the influence of the measuring error.

Another finding was that with increasing distance between the marking balls, the accuracy also increased.

Table 7-7. Peak value of the maximal error of the direction of the helical axes (Max. δ max.) for the three series A, B en C combined.

$\alpha_{nom.}$	Max. δ max.
0.1°	not determined
0.3°	55°
1.0°	14.17°
3.0°	4.67°
10.0°	1.53°
30.0°	0.63°

From this, the following equation may be deduced:

$$\delta_{max} = C \cdot \frac{V\ (mm)}{A\ (mm)} \cdot \frac{1}{\alpha_{nom}} \text{........ equation (1)}$$

in which δ_{max} is the maximal deviation of the directional vector, C is a constant, V is the variation or the maximal deviation of the spatial coordinates of the marking balls, A is the distance between the marking balls and α_{nom} is the nominal rotation angle.
Equation (1) is written differently as:

$$C = \delta_{max} \cdot \frac{A}{V} \cdot \alpha_{nom} \text{................equation (2).}$$

Now if we complete the above equation (2) with the five maximal values of δ in Table 7-7, we find:

$$C = 55 \cdot \frac{100}{0.1} \cdot 0.3 = 16500$$

$$C = 14.17 \cdot \frac{100}{0.1} \cdot 1 = 14170$$

$$C = 4.7 \cdot \frac{100}{0.1} \cdot 3 = 14100$$

$$C = 1.53 \cdot \frac{100}{0.1} \cdot 10 = 15300$$

$$C = 0.63 \cdot \frac{100}{0.1} \cdot 30 = 18900.$$

The mean of these five values of C amounts to 15794. Equation (1) may now be written as:

$$\delta_{max} = 15794 \cdot \frac{V}{A} \cdot \frac{1}{\alpha_{nom}} \text{.......equation (3).}$$

Apart from the maximal deviations, the mean values of δ were also computed for the first series of initial positions A. The mean values found are listed in Table 7.8, together with the maximal values.

Table 7-8. Maximal errors of the directions of the helical axes (δ max.), mean errors ($\bar{\delta}$) and quotient $\bar{\delta} : \delta$ max. for series A, first time (A1), for 6 different nominal rotation angles (α nom).

α nom	δ max.	$\bar{\delta}$	$\bar{\delta} : \delta$ max.
0.1°	71.0 °	-	-
0.3°	44.0 °	24.14°	0.5486363
1.0°	8.7 °	6.26°	0.7195402
3.0°	3.9 °	2.24°	0.5743582
10.0°	1.37°	0.79°	0.5766423
30.0°	0.4 °	0.22°	0.55
			mean 0.59

From these, now, the following equation can be deduced:

$$\bar{\delta} = 0.59\delta_{max} \text{ .... equation (4).}$$

Subsequently, the equations established could be applied to estimate, from the deviation of the computed helical axis of the HMTA, the deviation from the computed spatial coordinates that caused it.
The results of the HMTA computations are summarized in Tables 7-1 and 7-2. On the basis of Table 7-1 it can be computed that, were the upper combination is concerned, with a step size of 5° the δ_{max} is 1.58° and the $\bar{\delta}$ 0.86°. This gives us a ratio $\bar{\delta} : \delta_{max}$ of 0.55, in good agreement with equation (4).
In the HMTA, the marking balls were 18.5 mm apart. Equation (1) is now written as:

$$V = \frac{\delta_{max} \,.\, A \,.\, \alpha_{nom}}{C} \text{ .... equation (5).}$$

or

$$V = \frac{1.58 \,.\, 18.5}{15794} = 0.01 \text{ mm.}$$

Subsequently, the same procedure was followed for the HMTA data with step sizes of 10°, 15° and for the mean δ with $\alpha = 5°$ and $\alpha = 15°$. Ultimately, this procedure was applied to the lower combination, as well. The results obtained are listed in Table 7-9. For each nominal rotation α, this table each time shows the δ_{max}, the $\bar{\delta}$ and the ratio $\bar{\delta} : \delta_{max}$.
In addition, with the aid of equation (5), we always computed the variation of a maximal deviation error of the direction of the axis, and, with the aid of equation (4), the variation for a mean deviation. The table shows that a variation of the accuracy of the computation of the positions of the balls, ranging from 0.01 mm to a maximum of 0.015 mm, is to be expected.

Table 7-9. Results errors (δ) of the directional vector (φ : 90°), with the HMTA, 3 respective nominal rotation angles for upper and lower combination.

Upper combination			
Phase	α nom. 5°	α nom. 10°	α nom. 15°
	δ	δ	δ
1	0.52	0.08	0.17
2	0.37	0.45	0.24
3	0.57	0.33	0.53
4	1.13	0.82	0.58
5	1.58	1.26	0.69
6	1.52	0.72	
7	0.44		
	$\delta_{max.} = 1.58$	$\delta_{max.} = 1.26$	$\delta_{max.} = 0.69$
	$\bar{\delta} = 0.87$	$\bar{\delta} = 0.61$	$\bar{\delta} = 0.44$
	$\frac{\bar{\delta}}{\delta_{max.}} = 0.55$	$\frac{\bar{\delta}}{\delta_{max.}} = 0.48$	$\frac{\bar{\delta}}{\delta_{max.}} = 0.64$
	Var. = 0.009 mm by $\delta_{max.}$	Var. = 0.014 mm by $\delta_{max.}$	Var. = 0.012 mm by $\delta_{max.}$
	Var. = 0.009 mm by $\bar{\delta}$	Var. = 0.012 mm by $\bar{\delta}$	Var. = 0.013 mm by $\bar{\delta}$

Lower combination			
Phase	α nom. 5°	α nom. 10°	α nom. 15°
	δ	δ	δ
1	0.81	0.55	0.35
2	0.47	0.64	0.21
3	1.08	0.33	0.29
4	0.71	0.23	0.13
5	0.83	0.52	0.38
6	0.40	0.13	
7	0.47		
	$\delta_{max.} = 1.08$	$\delta_{max.} = 0.64$	$\delta_{max.} = 0.38$
	$\bar{\delta} = 0.68$	$\bar{\delta} = 0.40$	$\bar{\delta} = 0.27$
	$\frac{\bar{\delta}}{\delta_{max.}} = 0.63$	$\frac{\bar{\delta}}{\delta_{max.}} = 0.62$	$\frac{\bar{\delta}}{\delta_{max.}} = 0.71$
	Var. = 0.006 mm by $\delta_{max.}$	Var. = 0.008 mm by $\delta_{max.}$	Var. = 0.007 mm by $\delta_{max.}$
	Var. = 0.007 mm by $\bar{\delta}$	Var. = 0.008 mm by $\bar{\delta}$	Var. = 0.008 mm by $\bar{\delta}$

Aplication of equation (1) to the foot-lower leg preparations, given a distance of 18.5 mm between the marking balls, gives the following estimated mean and maximal deviations for the computed axial directions (see Table 7-10).

Table 7-10. Maximal errors (δ_{max}.) and mean errors ($\overline{\delta}$) of the direction of inclination in a comparable tarsal situation.

Nominal rotation angle α nom.	δ_{max} for an error in the calculated coördinates of 0.01 - 0.015 mm	$\overline{\delta}$ for an error in the calculated coördinates of 0.01 - 0.015 mm
1.5°	5.8° - 8.8°	3.3° - 5.0°
3°	2.9° - 4.4°	1.7° - 2.5°
4°	2.2° - 3.3°	1.2° - 1.9°
5°	1.8° - 2.6°	1.0° - 1.5°
7.5°	1.2° - 1.8°	0.7° - 1.0°
10°	0.9° - 1.3°	0.5° - 0.7°
15°	0.6° - 0.9°	0.3° - 0.5°

The maximal values (δ_{max}) and mean values ($\overline{\delta}$) computed in Table 7-10 correspond to a series of (selected) normal rotation angles in a situation comparable with that in the series of tarsal experiments.
The following survey, Table 7-11, shows the frequency of computed values of α for the relative movements TACA, CUCA, NACA and TANA that were smaller than 3°, between 3° and 5° and larger than 5°, for step 1 and for the steps 2 up to and including 5 or 6 together at 10° tibial laterorotation.

Table 7-11. Frequency of occurrence $\alpha<3°$, $3°<\alpha<5°$ and $\alpha>5°$ in tarsal helical rotations, step size 10°.

	$\alpha<3°$	α3°-5° incl.	$\alpha>5°$
TACA-1	3	3	4
TACA-2-5/6	0	7 (14.9%)	40 (95.1%)
CUCA-1	5	4	1
CUCA-2-5/6	0	20 (42.6%)	27 (57.4%)
NACA-1	5	3	2
NACA-2-5/6	0	8 (17.0%)	39 (83.0%)
TANA-1	0	2	8
TANA-2-5/6	0	0	47 (100%)
Total step 1	13 (32.5%)	12 (30.0%)	15 (37.5%)
Total step 2-5/6	0	35 (18.6%)	153 (81.4%)

From these data, we can deduce that given steps of 10° and omitting step 1, the mean deviations of the directions of the helical axes will amount to approx 2° in 18.6% of the steps. One-half of the estimated number of deviations occurs in CUCA, and approx one-quarter in TACA and NACA each. In all other cases, the estimated deviation is 1° or less. When, on the other hand, for our computations we should use the values corresponding to a tibial laterorotation in steps of 5°, we would find that for steps 2 to 6 or 7 inclusive, the number of rotations around the helical axes of TACA, CUCA, NACA and TANA smaller than 3° would increase from 0 to 34.1%, while 38.5% would be in the category from 3° to 5° inclusive and only 27.4% would be larger than 5° (Table 7-12).

Table 7-12. Frequency of occurrence $\alpha<3°$, $3°<\alpha<5°$ and $\alpha>5°$ in tarsal helical rotations, step size 5°.

	$\alpha < 3°$	$3° < \alpha < 5°$	$\alpha > 5°$
TACA-1	10	0	0
TACA-2-6/7	18	27	12
CUCA-1	10	0	0
CUCA-2-6/7	37	20	0
NACA-1	9	1	0
NACA-2-6/7	22	30	5
TANA-1	7	3	0
TANA-2-6/7	2	10	45
Total step 1	36 (90%)	4 (10%)	0 (0%)
Total step 2-6/7	77 (34.1 %)	87 (38.5 %)	62 (27.4%)