

ERRORS IN X-RAY PHOTOGRAMMETRY

6.1 INTRODUCTION

This analysis of errors begins with a survey of the various factors that may contribute to the occurrence of errors.

Although the individual contributions of these factors to the errors could not be determined, specially designed testing equipment and a computation model rendered it possible to gain insight into the influence of the errors on the X-ray photogrammetric data.

We have divided the errors into two groups:

- a. sources of error in the experimental set-up; these resulted in systematic errors (Chapter 6.2);
- b. sources of error in the measuring of X-ray photographs. These resulted in incidental errors (Chapter 6.3).

In addition, the systematic errors of the set-up are affected by incidental errors because for the determination of the position of the roentgen sources the projected balls of the source-locating apparatus had to be measured.

Chapter 7 deals with the errors in the data processing, and the interaction between these errors and those mentioned above.

Systematic errors in the set-up will affect the computed coordinates of all marking balls. This manifests itself within a deviation of the 6 (unalterable) distances between the 4 marking balls, as the result of which the object measured had undergone an apparent distortion. The actual error of each of the individual positions remains unknown, however. An impression of the extent to which errors occur, and of their magnitude may be gained with the aid of a number of parameters. These are:

- 1) the distance between the crossing lines at the locating of sources B_1 and B_2 (should in theory be 0);
- 2) the distance between the crossing lines at computation of the positions of each of the marking balls (should in theory be 0);

- 3) the six fixed distances between the 4 marking balls per bone (should in theory remain unchanged). Only two distances are known.

During all experiments, the above-named parameters have been computed, which rendered possible an adequate check on the reliability of the X-ray photogrammetric data. This was of great importance with a view to the computation of the helical axes subsequently to be performed. The parameters mentioned above were also computed for the test methods for which use was made of two calibration objects, the helical movement testing apparatus (HMTA) and the measuring plate (see description par 6.3). The reliability depended on whether or not standard values were exceeded for 1), 2) and 3); these standard values amounted to 1 mm, 0.1 mm and 0.1 mm respectively. These standard values have been established on the basis of the comparison of the combined, experimental and model analysis of errors (see Chapter 7). If standard values were exceeded, the measurements were repeated; if this once more failed to provide a good result, the series was excluded from the subsequent computation procedure. (This occurred only once.)

6.2 SOURCES OF ERROR IN EXPERIMENTAL SET-UP (SYSTEMATIC ERRORS)

Errors in the experimental set-up may be caused by:

- a. Deviations from the ideal planarity of the X-ray cassette against which the X-ray films are pressed, resulting in a less than ideally planar 'photographic plane'.
- b. The angle between the cassettes in the cage deviating from the ideal 90° exactly.
- c. Errors in the measuring, by means of measuring clock and slide rule of the fixed distances in the cage (see also Appendix A) and of the source-locating apparatus. The cage dimensions in question are: XLVK = 96.5 mm; ZSVK = 108.00 mm; XLHK = 95.5 mm and YSHK = 19.5 mm. Then there are the distances of the horizontal and vertical X-ray photoplane from the origin O_s of the cage. These provided an added, constant third coordinate, respectively YVF = -12.5 mm; ZHF = -23.9 mm (see also Appendix A). Finally, the measured fixed distances on the source-locating apparatus. These are the coordinates of P_{1s} , P_{2s} and P_{3s} (see Chapter 5 and Table 6-1).

Table 6-1. Coördinates of the focus locating apparatus (FLA).

| | X | Y | Z |
|------------|--------|--------|--------|
| P_{1s} : | 62.00 | 14.38 | 131.03 |
| P_{2s} : | 122.78 | 132.00 | 131.03 |
| P_{3s} : | 22.00 | 150.00 | 42.06 |

- d. The positioning of the focus-locating apparatus in the cage against the X- and Y-ribs. N.B.: The floor plate of the FLA was made of perspex with a relatively high independence from temperature and humidity.
- e. Use of X-ray sources not ideally rigidly fixed in relation to the cage.
- f. The non-punctiform shape of the source of the roentgen tubes: the Practix and Standard have a focus surface of approx 2 sq. mm! According to Philips, the surface measures 1 sq.mm; the intensity at the edge of the radiating surface exceeds that in the centre; this causes a blurring that is corresponding to an anode with a surface of 2 sq.mm.

6.3 ERRORS MADE IN MEASURING X-RAY PHOTOGRAPHS (ACCIDENTAL ERRORS)

By measuring a random ball a number of times, an impression may be gained of the error that may occur during the measuring of an X-ray photograph with the comparator. Table 6-2 lists the results of 20 measurements in the horizontal photograph of the projection coordinates (the so-called 'photo coordinates') of measuring ball nr 23.

Table 6-2. Coördinates random ball nr. 23. Iterative measurements on comparator. Photograph nr. 3 (horizontal projection).

| N | | | N | | |
|------------------------|----------------------|----------------------|------------------------|----------------------|----------------------|
| Number of measurements | X-coörd. ball centre | Y coörd. ball centre | Number of measurements | X coörd. ball centre | Y coörd. ball centre |
| 1 | 782.978 | 781.835 | 1 | 782.973 | 781.847 |
| 2 | 782.990 | 781.823 | 2 | 782.975 | 781.845 |
| 3 | 782.978 | 781.842 | 3 | 782.977 | 781.842 |
| 4 | 782.982 | 781.852 | 4 | 782.985 | 781.842 |
| 5 | 782.982 | 781.837 | 5 | 782.985 | 781.845 |
| 6 | 782.978 | 781.847 | 6 | 782.983 | 781.860 |
| 7 | 782.982 | 781.852 | 7 | 782.977 | 781.848 |
| 8 | 782.978 | 781.847 | 8 | 782.978 | 781.848 |
| 9 | 782.992 | 781.838 | 9 | 782.978 | 781.842 |
| 10 | 782.977 | 781.843 | 10 | 782.982 | 781.853 |

The mean value of these 20 X-coordinates was 782.981 mm, standard deviation \pm 0.0048 mm; range 782.973 mm to 782.992 mm. (0.019)

The mean value of these 20 Y-coordinates was 781.844 mm, standard deviation \pm 0.0078 mm; range 781.823 mm to 781.860 mm. (0.037)

Interestingly, this shows that the error in the Y-coordinate is larger than the error in the X-coordinate, even though there is only a difference between the two ranges of 0.018 mm! The direction of the Y-axis in the photograph paralleled the direction of the X-axis of the comparator; this was the horizontal direction in the measuring site. One explanation of this difference is that the margins of the projected balls in the horizontal photograph are minimally blurred in the (negative) X-direction owing to the oblique dorsoplantar projection.

6.4. TESTING RESULTS OF X-RAY PHOTOGRAMMETRY

Test methods to study the influence of errors on the X-ray photogrammetric data.

For testing purposes we have devised two calibration objects, the so-called HMTA and the *measuring plate*.

6.4.1 The HMTA (Helical Movement Testing Apparatus)

The HMTA (Helical Movement Testing Apparatus) was designed with a two-fold purpose, viz. to study the influence of errors on the X-ray photo-grammetric data and the influence exerted by errors in these data on the helical movement (ultimately) to be computed. This second use is described in Chapter 7.

The HMTA comprises a brass cylinder which on a base can rotate around a fixed vertical longitudinal axis (see Figure 6-1). The base carries a stepping mechanism constructed in such a way that the cylinder can be rotated 5° at a time. Attached to the cylinder is a perspex rod that carries four pins in two pairs. These pins are of the same type that is used for the tarsal bones, each pin encasing two marking balls. Accordingly, the HMTA carries 8 marking balls in all, each capable of rotating in a horizontal plane. Prior to our study, the angles of rotation of the HMTA had been measured with optical equipment in the Kamerlingh Onnes Laboratory. The results of these measurements are listed in Tables 6-2 and 6-3.

Table 6-3. Helical movements testing apparatus optically measured values of rotations (α_{Om}) and their mean values ($\bar{\alpha}_{Om}$).

| Transition positions | α_{Om} | Transition positions | α_{Om} | Transition positions | α_{Om} | $\bar{\alpha}_{Om}$ |
|----------------------|---------------|----------------------|---------------|----------------------|---------------|---------------------|
| 0- 5 | 4.98° | 0- 5 | 5.00° | 0- 5 | 5.01° | 4.998° |
| 5-10 | 4.97° | 5-10 | 4.97° | 5-10 | 4.97° | 4.970° |
| 10-15 | 4.96° | 10-15 | 4.97° | 10-15 | 4.97° | 4.968° |
| 15-20 | 5.02° | 15-20 | 5.00° | 15-20 | 5.01° | 5.011° |
| 20-25 | 4.97° | 20-25 | 4.99° | 20-25 | 4.99° | 4.981° |
| 25-30 | 5.02° | 25-30 | 5.01° | 25-30 | 5.02° | 5.017° |
| 30-35 | 4.98° | 30-35 | 4.98° | 30-35 | 4.99° | 4.985° |

Table 6-4. Optically measured values of rotations (α_{om}), maximal and minimal values, differences (Max. α_{om} and Min. α_{om}).

| Transition positions | Min. α_{om} . | Max. α_{om} | Difference (Max. α_{om} and Min. α_{om}). |
|----------------------|----------------------|--------------------|--|
| 0- 5 | 4.98° | 5.01° | 0.03° |
| 5-10 | 4.97° | 4.97° | 0.00° |
| 10-15 | 4.96° | 4.97° | 0.01° |
| 15-20 | 5.00° | 5.02° | 0.02° |
| 20-25 | 4.97° | 4.99° | 0.02° |
| 25-30 | 5.01° | 5.02° | 0.01° |
| 30-35 | 4.98° | 4.99° | 0.01° |

In three measuring series of 7 steps, the range of the variation (see Table 6-3) of α_{om} was 0.01° to 0.03°, with a mean variation of 0.014°. These deviations are believed to be caused by errors in the optical measuring methods and imperfections in the construction of the HMTA. During tests, the HMTA is positioned in the experimental set-up in such a way that the projections of its marking balls lie approximately in the projection area of the tarsal marking balls. Then, using sources B_1 and B_2 , X-rays are made of the eight rotation positions (5° at a time, 0 to 35°) and the source-locating procedure is carried out. After the measuring, the coordinates were computed.

It was our intention to use the test equipment to find an answer to the following three questions:

- 1) What accuracy can be reached in determining the position of a marking ball in the experimental set-up?
- 2) Is the determination of the position more accurate when the marking ball is situated closer to the origin, or in other words: do differences exist for the marking balls of the upper and the lower combination?
- 3) Does an increased distance between the two marking balls improve the accuracy?

In regard to the results, attention was paid to:

- 1) The length of the interlinear distance between the crossing lines at source-locating of B_1 and B_2 (actually therefore, just a check on the estimating of DLVC and DSVC together with the accuracy of the cage and of the source-locating apparatus).
- 2) The degree of constancy of the Z-coordinates of the measuring balls of the HMTA moving in eight different horizontal planes.
- 3) The length of the interlinear distance between the crossing lines at determination of the positions of each HMTA marking ball.
- 4) The constancy of the 6 distances for the two combinations of four HMTA marking balls.

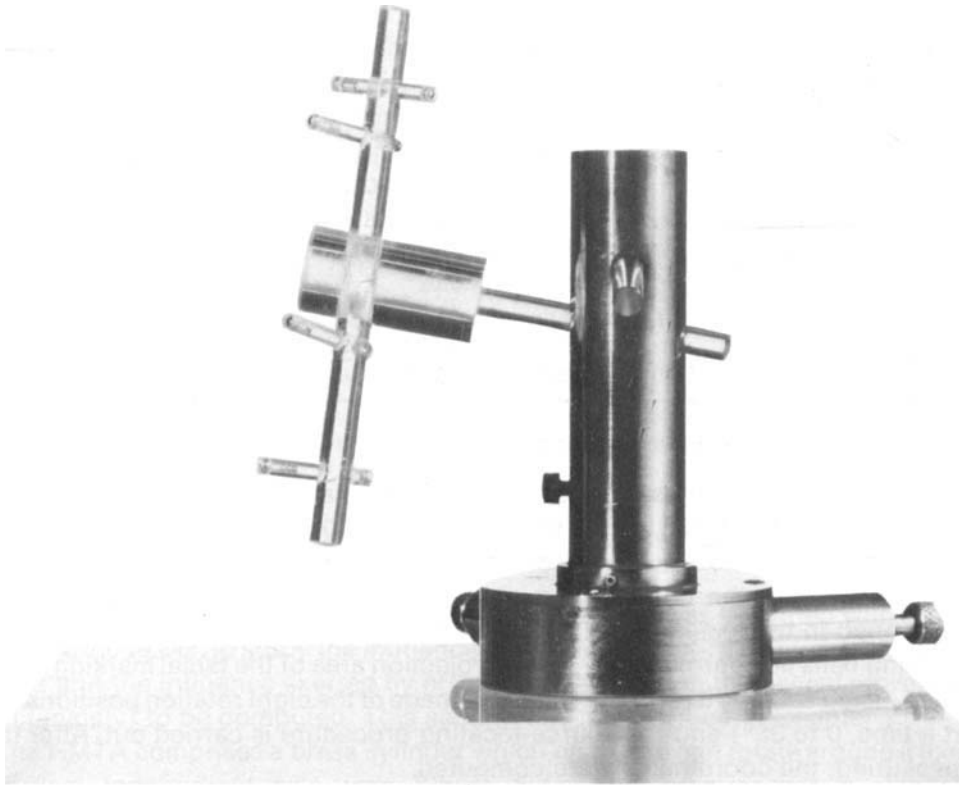


Figure 6-1. Helical Movement Testing Apparatus (HMTA).

All tests were performed by student's method for two random samples. All tests were twosided, at a 0.05 level of significance.

sub 1)

Interlinear distance of the crossing lines at source location B_1 ; these values are listed in Table A-1 in the Appendix, first on the basis of estimates of DLVC and DSVC and subsequently on the basis of two repetitive computations. For a 'distance' of approx 1143 mm, the ultimate interlinear distance amounted to 0.17 mm.

The interlinear distance of the crossing lines at source location B_2 is also listed in this table, in the same way as for B_1 . For a distance of approx 1.287 mm, this distance now amounted to 0.72 mm.

Here, again, we find that the interlinear distance for source B_2 is longer than that for source B_1 , and that a difference in the deviations of the reference points exerts a greater influence on the interlinear distance of source B_2 than on that of source B_1 .

sub 2)

Degree of constancy of Z-coordinates of HMTA balls (see Table A-2, A-3 Appendix).

For the various values of the Z-coordinates of the upper combination of marking balls P_1 , P_2 , P_3 and P_4 a maximal difference of 0.05 mm was found. For the lower combination, we also determined the maximal difference for the Z-coordinate for P_1 , P_2 , P_3 and P_4 , respectively. The maximal difference for the Z-coordinates of these lower four balls amounted to 0.07 mm.

It may be concluded that there is no significant difference ($p = 0.43$) between the upper and the lower combination; in other words, the distance of the balls from the origin does not affect the accuracy of the Z-coordinates computed.

sub 3)

Interlinear distance of crossing lines at computation of position of HMTA marking ball.

The distance between the crossing lines B_1P_{nv} and B_2P_{nh} was calculated for each ball in each position. Table A-2 in the Appendix lists these distances for the combination of the 'upper' four marking balls.

For ball P_1 , the mean value calculated for all positions was 0.09 mm, the highest value was 0.13 mm, the lowest value 0.05 mm. For ball P_2 the mean value was 0.10 mm with a maximum of 0.14 mm and a minimum of 0.06 mm. Ball P_3 : mean 0.10 mm, maximum 0.14 mm and minimum 0.05 mm, and ball P_4 : mean 0.10 mm, with a maximum of 0.14 mm and a minimum of 0.05 mm. For the combination of the four upper balls, in eight positions, the interlinear distance $B_1P_{nv} - B_2P_{nh}$ had a mean value of 0.10 mm (0.095 mm). The standard deviation amounted to 0.024 mm. Table A-3 (Appendix) lists the distances for the combination of the lower four marking balls. The mean distances amounted to 0.06 mm for ball 1, 0.07 mm for ball 2, 0.07 mm for ball 3 and 0.06 mm for ball 4.

For the combination of the lower four balls, in eight positions, the mean interlinear distance $B_1P_{nv} - B_2P_{nh}$ was 0.06 mm. The standard deviation was 0.025 mm.

Accordingly, the mean value of the interlinear distance for the lower combination of balls is significantly smaller than that for the upper combination ($p < 0.001$), although the difference amounts to only 0.04 mm. This may be explained by the degree of accuracy that could be achieved in constructing the cage. Clearly, the accuracy possible for the computation of balls situated close to the origin is greater than that for balls farther from the origin.

sub 4)

Degree of constancy of six fixed distances between balls.

Tables A-2 and A-3 (Appendix) present a survey of the six distances for the 'upper' and 'lower' combination of four marking balls.

Since the marking balls in principle do not move in relation to each other, each of these six fixed distances, in all eight positions, will have to remain unchanged. Table 6-5 shows that the variation in the distances between the HMTA balls

ranges from 0.06% to 0.17% (mean 1‰) for the upper balls and from 0.1% to 0.22% for the lower balls (mean 1.5‰). In addition, this table shows that with increasing distance between the balls, the degree of accuracy achieved does not always increase. The table also shows that when the balls are situated closer to the origin (the balls of the lower combination), the accuracy accomplished is not greater than that for balls situated farther from the origin.

Table 6-5. Mean values and mean errors in distances H M T A balls (upper and lower combination).

| H M T A Upper combination | | | H M T A Lower combination | | |
|-------------------------------|---------------|---------------|-------------------------------|---------------|---------------|
| Distance | Mean n = 6 | Mean error | Distance | Mean n = 6 | Mean error |
| P ₁ P ₂ | 17.37 mm | 0.17% | P ₂ P ₄ | 12.01 mm | 0.17% |
| P ₃ P ₄ | 17.57 mm | 0.06% | P ₁ P ₄ | 13.69 mm | 0.22% |
| P ₁ P ₄ | 19.85 mm | 0.15% | P ₁ P ₃ | 15.72 mm | 0.19% |
| P ₂ P ₃ | 22.48 mm | 0.09% | P ₁ P ₂ | 17.59 mm | 0.11% |
| P ₁ P ₃ | 25.69 mm | 0.08% | P ₃ P ₄ | 17.67 mm | 0.11% |
| P ₂ P ₄ | 28.26 mm | 0.07% | P ₂ P ₃ | 20.49 mm | 0.10% |

6.4.2 The measuring plate

Since the position of the balls in the HMTA always undergo only small changes, the influence of the systematic errors also remains little. The accuracy of the determination of the helical axis depends on the magnitudes of changes of position. Now in order to gain some impression of this systematic error, greater changes of position of balls were studied. To this purpose, the measuring plate was elaborated. It is a perspex plate measuring approx 80 mm x 80 mm and approx 10 mm thick, with at the corners marking balls approx 1 mm in diameter; these marking balls differ from the type used in the tarsus and in the HMTA. The dimensions of the measuring plate were chosen intentionally; the coordinates of the measuring plate balls more or less 'delimite' the area in which the tarsal marking balls, also, will move.

Table 6-6. Values X, Y and Z coördinates of measuring plate balls, maximal and minimal.

| |
|-----------------------------|
| X: 38.75 Min. - 122.72 Max. |
| Y: 129.2 Min. - 91.28 Max. |
| Z: 10.82 Min. - 92.65 Max. |

The experiment with the measuring plate was carried out in a manner analogous to that used in the HMTA. The measuring plate could be placed in six different positions, two different positions in each of three planes, viz in a plane approximately perpendicular to, respectively, the X-axis (positions 1 and 2), the Y-axis (positions 3 and 4) and the Z-axis (positions 5 and 6). For the six positions of these four measuring plate balls we again calculated the X, Y and Z coordinates and the six fixed distances between the four balls (see Table 4 Appendix). For each of these distances we have computed a mean, a standard deviation, a relative error and an absolute error. These data are listed in Table 6-7. The table shows that the maximal absolute error in this experiment amounted to 0.18 mm for a distance P_2P_4 of 106.2 mm, with a relative error of 2‰. The smallest absolute error in this experiment was 0.03 mm, for a distance P_1P_4 of 96.90 mm and the relative error was 0.4‰.

Table 6-7. Mean values, standard deviations (SD) and mean errors in distance measuring plate balls.

| Distance | Mean (mm) n = 6 | SD (mm) | Rel. error | Abs. error (mm) |
|----------|--------------------|---------|------------|--------------------|
| P_1P_2 | 77.15 | 0.04 | 0.001 | 0.08 |
| P_1P_3 | 105.99 | 0.08 | 0.002 | 0.16 |
| P_1P_4 | 76.90 | 0.02 | 0.0004 | 0.03 |
| P_2P_3 | 76.97 | 0.05 | 0.001 | 0.10 |
| P_2P_4 | 106.20 | 0.09 | 0.002 | 0.18 |
| P_3P_4 | 77.18 | 0.07 | 0.002 | 0.14 |

Now if we compare the relative error of 2‰ corresponding to inter-ball distance P_2P_4 (106.2 mm) of the measuring plate with, for instance, the relative errors of the HMTA (see table 6-5, 1‰ and 1.5‰), we find that a shortened ball distance here causes no increase of the error.

These findings show that systematic errors leading to apparent deformation are negligible in respect to the standard imposed.