CHAPTER III

METHOD OF X-RAY PHOTOGRAMMETRY AND MOTION ANALYSIS OF TARSAL BONES

3.1 METHOD OF X-RAY PHOTOGRAMMETRY

The aim is to achieve an optimally exact description of the motions of the tarsal bones, as they take place during movements of the foot defined as accurately as possible. Since we assume that the tarsal bones do not move in relation to each other around fixed hinge axes, we have to use three-dimensional recording and analysis.

A movement may be recorded by selecting three points on the moving body and recording either the complete path of their movement or a number of selected points of this path.

These single points on the path may then be regarded as discrete positions of the body in its 'interrupted' movement. It has been demonstrated (Huson (1977) and Benink (1977)) that when the recorded points of a discontinuous and of a continuous movement are compared, the pathway found is largery the same.

As mentioned in Chapter 2, various investigators have used different methods to record the tarsal movements. We concluded that none of the methods mentioned was sufficiently accurate either qualitatively or quantitatively. In X-ray photogrammetry we have a useful method for the highly accurate recording of the positions of three-dimensional bodies. Hallert (1953) described X-ray photogrammetry as follows: "X-ray photogrammetry is the science of measurement with the aid of X-ray photographs of an object in order to determine primarily geometric qualities as size, position and shape of the object". X-ray photogrammetry is based on the concept of central projection. This means that the roentgen image of an object is formed by a bundle of X-rays which has its centre in the focus of the roentgen tube. The roentgen image is created where this bundle of rays intersects with a plane.

The method requires the adoption of certain postulates, viz:

1. that all X-rays originate from the same (mathematical) point;
2. that all X-rays travel in straight lines;
3. that the plane where the roentgen image is formed is an ideal plane.

In practice "these conditions will never become mathematically strict". We shall return to this subject in Chapter 6: 'analysis of errors in X-ray photogrammetry'.

With the aid of X-ray photogrammetry we were able in our experimental set-up exactly to calculate the spatial positions of the lower leg and of four tarsal bones during a number of successive phases of movement. In order to obtain sharp marking points in the X-rays of the tarsal bones perspex pins were implanted into the tibia, the fibula, the talus, the calcaneus and the navicular and cuboid bones, each pin containing two marking balls and one or several identification balls. For our study we used 4 fixed measuring points, two pins per bone. Twelve pins were implanted per foot-lower leg preparation: 2 into the tibia, 2 into the fibula, 2 into the calcaneus, 2 into the talus, 2 into the navicular and 2 into the cuboid bone; in all, 24 marking balls per preparation.

The first step consists of accurate recording of the projections of tarsal marking balls in two different directions. To this purpose, the foot-lower leg preparation was placed in a testing apparatus consisting of a cage with fixed reference points. Subsequently, the preparation went through an interrupted motion consisting of 6 or 7 successive stages ('steps'). After every step, X-rays were made in two directions. The several projections of the marking balls in the X-ray films were measured by means of a comparator.
Subsequently, from those various projections the spatial coordinates of the tarsal marking balls corresponding to each phase of the movement were computed.

3.2 METHOD OF COMPUTATION OF POSITIONS OF TARSAL MARKING BALLS

This is schematically represented in Figure 3.1.

From source B_1 the coordinates of which in relation to the origin have been predetermined, a tarsal marking ball (P_{ns}) is projected on the vertical X-ray films plane in point P_{nv}. The photographic coordinates of P_{nv} are measured with the aid of the comparator relative to the projected origin. Since the position of the projected origin in the vertical X-ray film relative to the actual origin O is known (see Appendix, Fig. 1), the photographic coordinates of P_{nv} in the X- and Z-directions may now be corrected to coordinates relative to O. To this, we add a Y-coordinate which is identical for all points P_n and has a negative sign. This, namely, is the distance between the vertical photographic plane and O. We now can compute a line that passes through point B_1 and point P_{nv}; this line would have to pass through point P_n. From source B_2 the same point P_n is projected on the horizontal roentgen plane in point P_{nh}. This, also, is measured and subsequently corrected so that then, the spatial coordinates relative to O of this point, also, are known. Subsequently, the line between B_2 and P_{nh} is computed and this, also, ought to pass through point P_n.

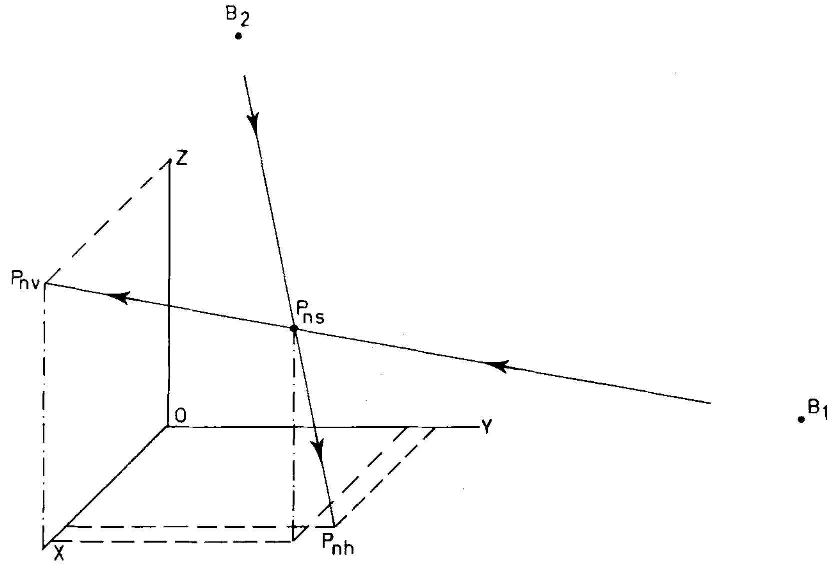

Figure 3-1. Scheme of computation of coördinates of spatial ball P_{ns} from the source coördinates B_1 and B_2 and the corresponding projection coördinates P_{nv} and P_{nh}.

Owing to incidental and systematic errors, the lines B_1P_{nv} and B_2P_{nh} will not intersect but cross. Therefore, the shortest distance between these crossing lines is calculated, and the point halfway is now regarded as the spatial position of P_n.
Thus, the photographic coordinates of the X-ray films described in Chapter 3.1. are processed to transfer them into coordinates with respect to the reference system of the cage. The spatial coordinates of the marking balls are computed (Chapter 3.2). Then, these spatial coordinates are fed into the helical movement computing programme, see Chapter 3.3.

3.3 METHOD OF MOTION ANALYSIS

Brief survey of the method of computing helical movements from spatial coordinates. This summary has been written in co-operation with C.W. Spoor and is essentially based upon the method described by Spoor and Veldpaus (1977).
The computation method makes use of the coordinates, determined by X-ray photogrammetry, of the marking balls in their starting and end positions, i.e. before and after each phase of the movement. By means of the mathematical procedure, the marking balls are now rotated and translated from their starting position in such a way that they approach the end position as closely as possible. It will not be possible to reach exactly the given end position of the marking balls

owing to inaccuracy in measuring and deformation of the bone. The imaginary helical axis around which the rotation, and along which the translation take place is regarded as the 'discrete' axis of movement of the corresponding phase of the movement. An optimal approximation of the helical movements means that the differences between the given coordinates of the end positions of the marking balls, and the coordinates obtained by the mathematical procedure are as small as possible. A widely used method to keep the difference minimal is keeping the sum of the squares of the differences as small as possible (the socalled smallest square method). Proceeding from 4 marking balls in a bone with 12 coordinates in all, the mathematical procedure will also supply 12 differences in coordinates, and the sum of the squares of these differences should be minimal. To this purpose, a standard has been introduced that is related to the accuracy of measurement; when this limit is exceeded, the computation process is stopped. The mathematical procedure can roughly be described as follows. A rotation may be described by a rotation matrix. This consist of an orderly system of 9 numerals which should fulfil certain conditions.

The rotation matrix R describes a rotation around an axis that passes through the origin. If in reality the rotation takes place around a parallel axis not passing through the origin, translation with a translation vector is required for correction. The translation is described by a translation vector that consists of 3 numerals. If a *helical* movement takes place around an axis through the origin, this will again be described by a rotation matrix with an added translation along that axis. However, if the helical movement takes place along an axis outside the origin, this movement admittedly is once more described by such a rotation matrix, but now, a correction translation has to be applied with added to it, a translation along the axis of rotation.

In regard to the rotation matrix and the (corrective) translation vector, the following should be added. The rotation matrix and translation vector together contain 12 unknown factors. The 12 unknown factors are the coordinates of the approximative end positions sought. These 12 unknown factors occur again in the sum of the squares of the differences of the coordinates.
The problem of determining these 12 unknown factors in such a manner that the sum of the squares of the differences is minimal and a number of conditions will be fulfilled as well, may be reduced to a system of 18 equations with 18 unknowns. These equations may be solved. We now know the rotation matrix R and the translation vector V. From these, the parameters of the helical movement have to be deduced.
From the rotation matrix we calculate the directions of the axis of rotation, the angle and the direction of the rotation. From the rotation matrix R and the translation V we now compute the position of the axis of rotation, and the translation along this axis.